Measurement and Modelling of Stiffness in Frictional Joints

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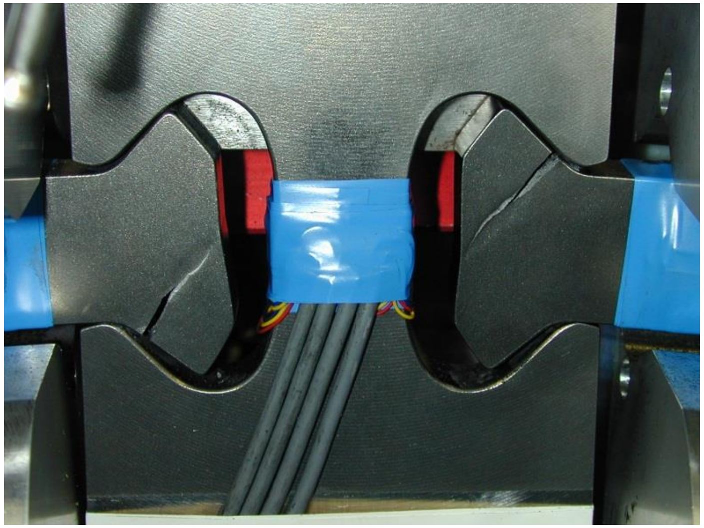
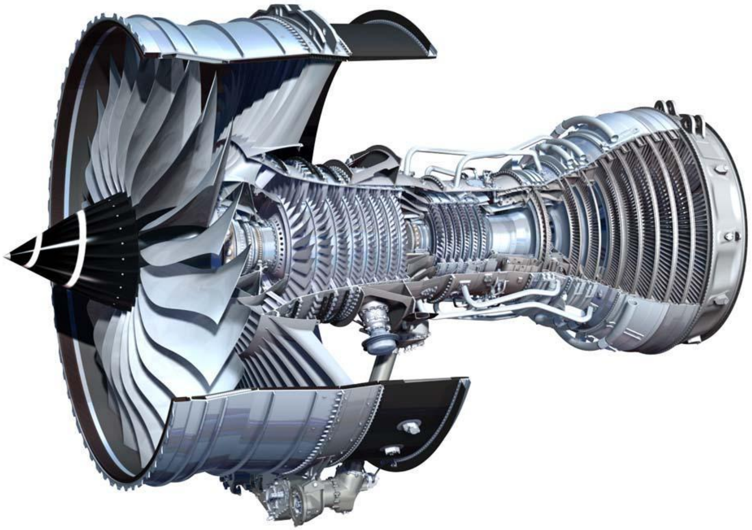
Background: Fretting Fatigue

- Fretting fatigue occurs when interfaces in a system are subjected to oscillatory forces or displacements
- The interface may be either
  - Sliding everywhere during part of the cycle
  - In partial slip, so that a portion of the contact stays 'stuck' during the complete cycle
- This oscillatory motion causes
  - Surface degradation
  - Possible changes in friction coefficient
  - Very high local stress (similar to a very sharp notch)
- Competing processes emerge
  - Crack nucleation
  - Small crack propagation
  - Wear



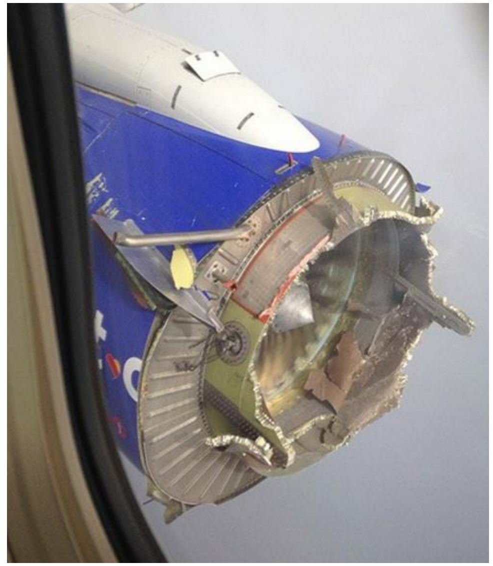
Why Interfaces?

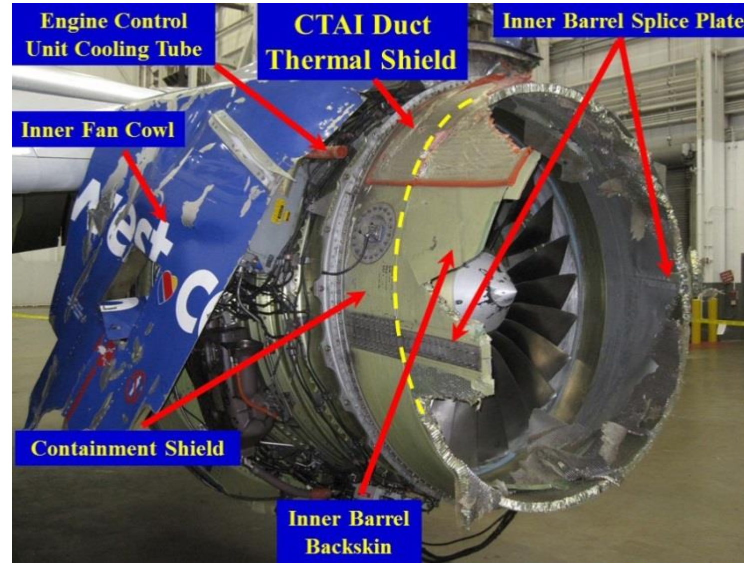
- So, interfaces in a system may be regarded as rather a nuisance
- But there are good reasons for having them:
- It is usually difficult to make the entire system from a single component
    - Restrictions on size of components
    - Restrictions on manufacture of particular geometries
    - Need for different materials in different parts of the system
    - Need for disassembly and replacement of parts in the system
    - Need to allow relative motion between components

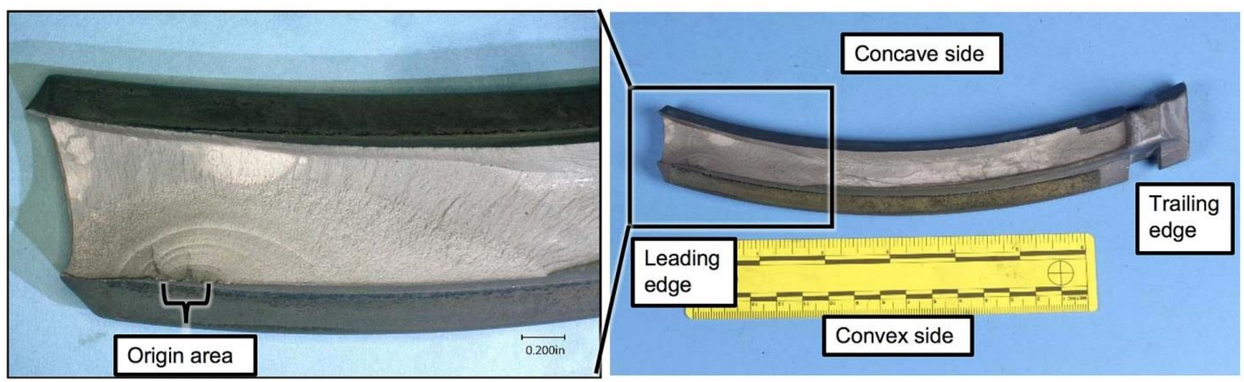


Fretting Fatigue in Practice

- South West Airlines flight 1380, 17th April 2018
- First fatality on US commercial flight since 2009
- NTSB report:
- A low-cycle fatigue crack in the dovetail of fan blade No. 13, which resulted in the fan blade separating in flight





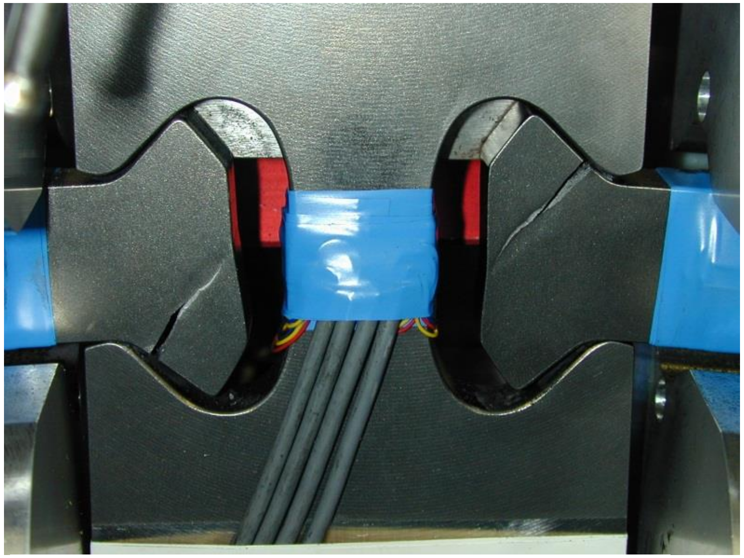


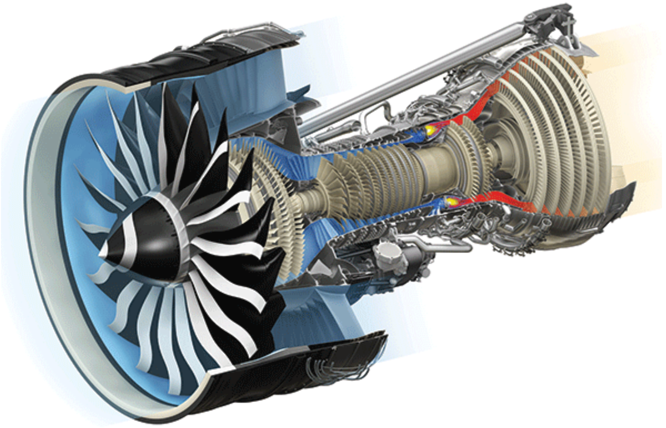
Fretting Fatigue in Practice

- Unfortunately, our current state of understanding does not prevent the occurrence of such incidents
- NTSB 2016:
- A low-cycle fatigue crack in the dovetail of fan blade No. 23, resulted in the fan blade separating in flight and impacting the fan case….

Solid Mechanics Requirements

- From a solid mechanics perspective a 'good' joint is one that has zero friction (we don't need to worry about it) or infinite friction (the two components are one)
- Unfortunately, from a vibrations perspective, we would like to see damping in our systems (i.e. mechanical energy turned into heat)
- In this second case a good system is one with neither zero nor infinite friction



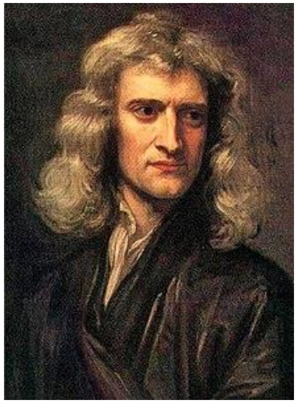


A Familiar Equation

- **This is probably the most important equation of applied mechanics**

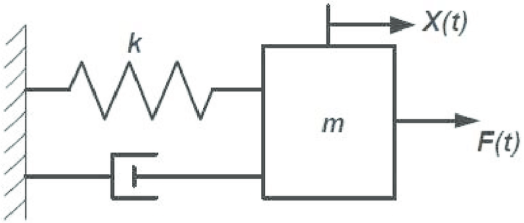
?

$$m\ddot{x} + c\dot{x} + kx = [\text{Periodic Force}]$$



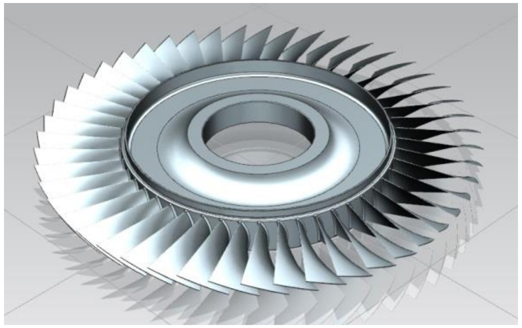
Isaac Newton
1543-1727

Robert Hooke
1535-1703



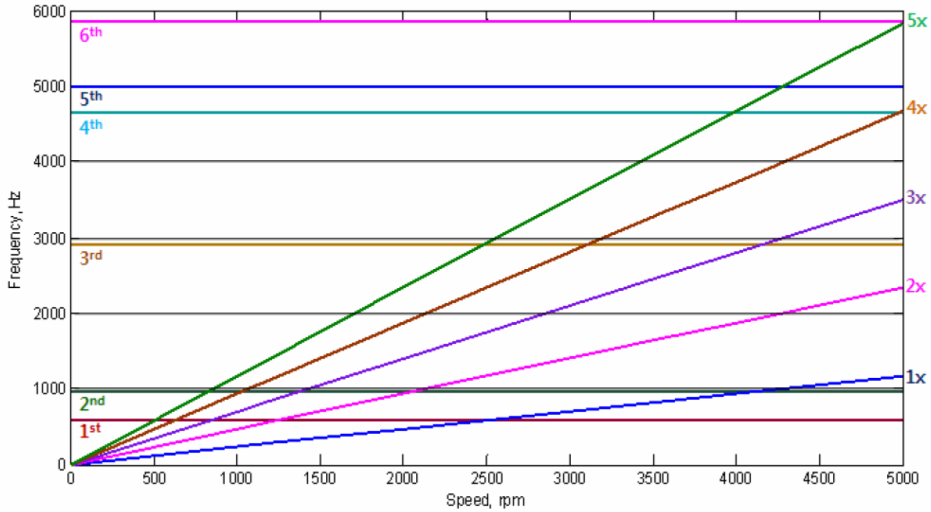
Frictional Damping

- Joints in a structure provide quite significant frictional damping
- This damping is highly non-linear
- Often it is simply fortuitous, but it can also be added deliberately
  - E.g. under-platform dampers

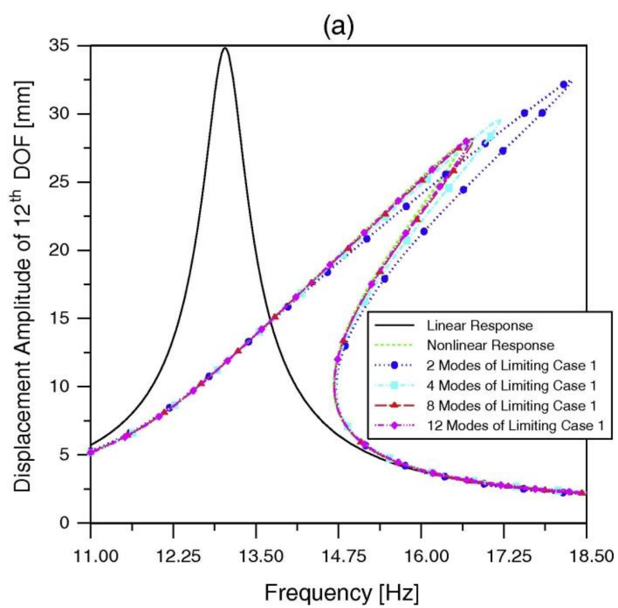


Vibration in Aeroengines

- The Campbell Diagram plots frequency against engine speed
  - Resonances may occur when a forcing frequency (multiple of engine speed) coincides with a blade response)
- Non-linearity means that the response may differ according to how the resonance is approached



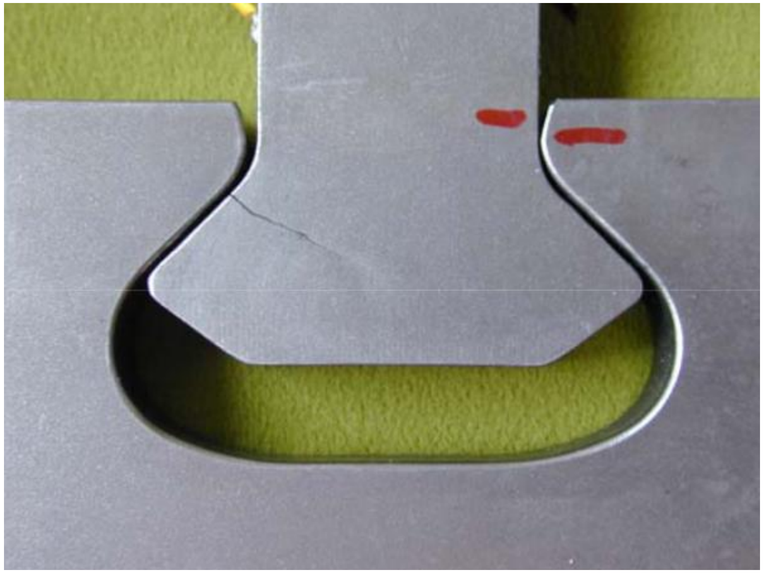
From Choi and Lee (2010)

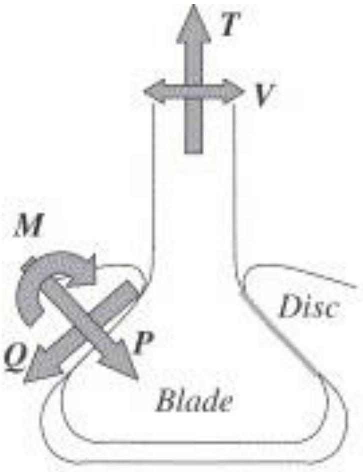


From Ferhatoglu et al (2018)

Analysis of Fretting Fatigue

- So, zero friction and infinite friction eliminate the fretting problem, but
  - Neither are realistically achievable
  - They each provide zero damping, so are the worst cases from the vibration perspective
- Analysis of fretting is also difficult
  - Non-linear
  - Rapidly varying stress
  - Size effect similar to that for notches
  - Unknown influence of surface damage
  - Cannot simply analyse by ‘hot spot’ stress on its own

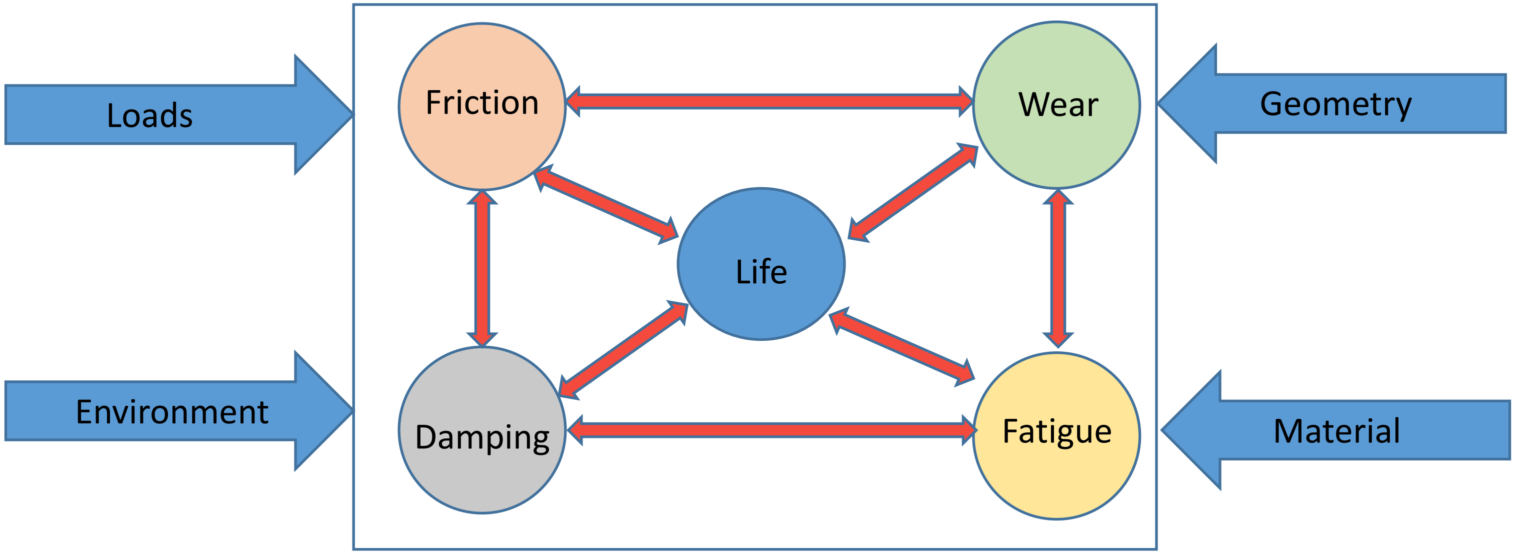




(a)

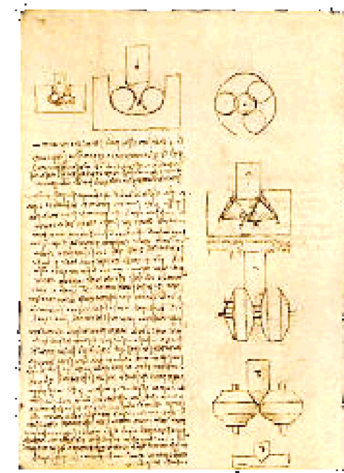
Are joints a good thing?

- Aside from the practical issues requiring joints, structures without frictional damping may be prone to large vibration amplitudes
- If we introduce joints then damping improves vibration performance, but potentially at the expense of
  - Fretting Fatigue
  - Fretting Wear
- Hence we need a holistic design approach which considers all the relevant parameters

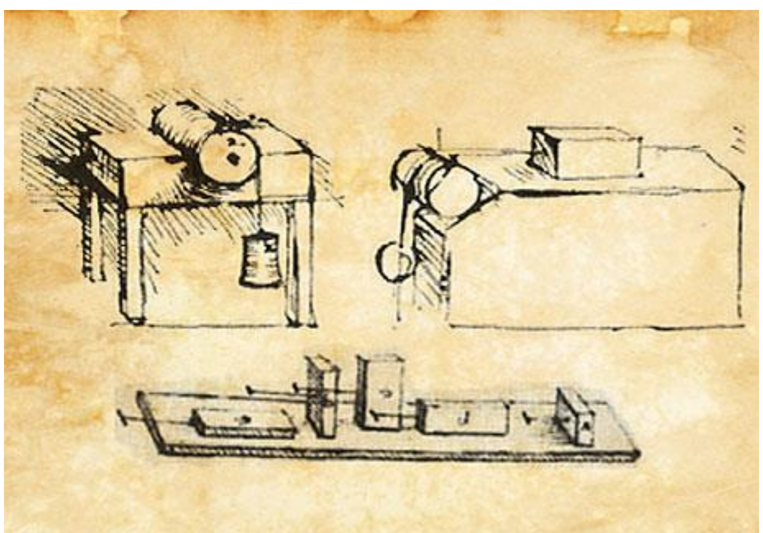


Friction Coefficient

- For many practical engineering purposes, the friction coefficient is assumed to be a constant for a material pair
    - This follows the basic understanding developed by Leonardo da Vinci 500 years ago
- But in experiments it can be observed that the coefficient varies with both time and position within a contact
- Moreover contacts are not repeatable, meaning that prediction can be challenging

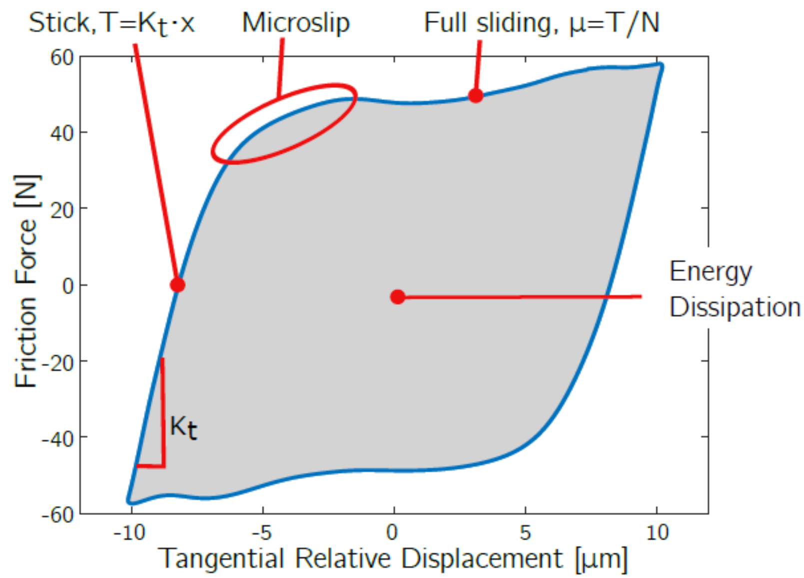


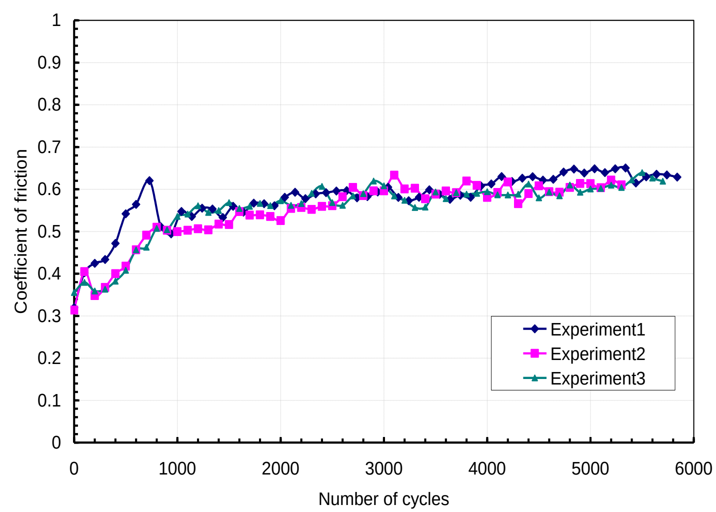
Leonardo da Vinci (1452 – 1519)

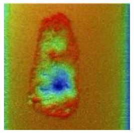


Friction Coefficient (2)

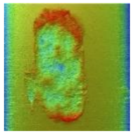
- Friction varies even during a simple cycle
- Reciprocating fretting tests show that surface damage plays a role in developing self-registration of surfaces
- As wear proceeds, friction can increase significantly



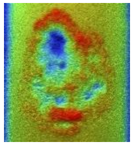




5 min

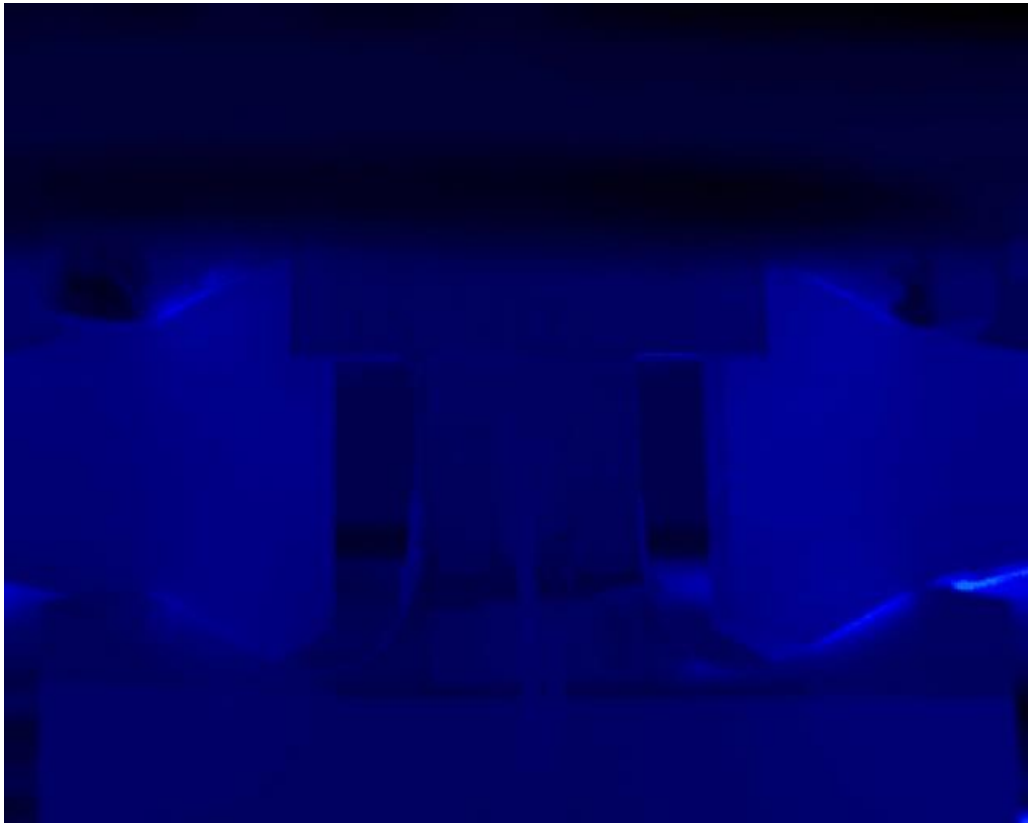


8 min



30 min

Variability in Friction



- Note how frictional heat generation differs at the four contacts

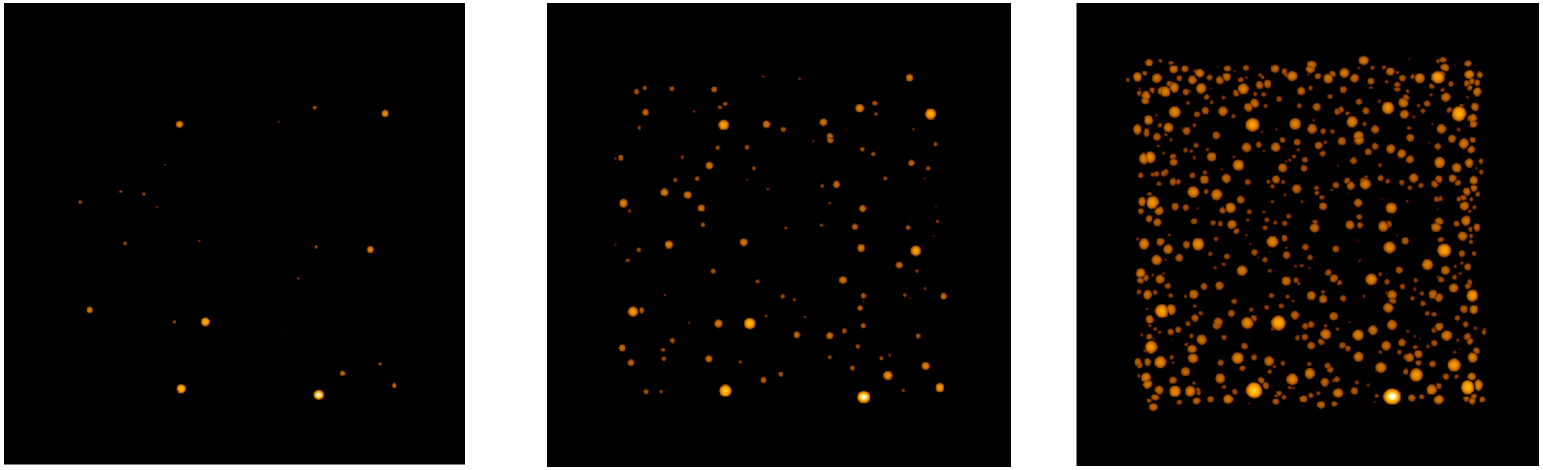
Where do we need to make progress?

- A ‘top down’ model is unlikely to be fully predictive
    - Phenomenologically-based
    - Will correlate only over a limited parameter space
    - May be computationally efficient
- A ‘physics-based’ model should be fully predictive
    - Based on understanding the joint behaviour in detail
        - Is this simply phenomenological at a lower length scale?
    - Should predict over a wide parameter space
    - Likely to be complex and difficult to use directly at the structural level

- Example – contact stiffness
    - What physics do we need to capture?

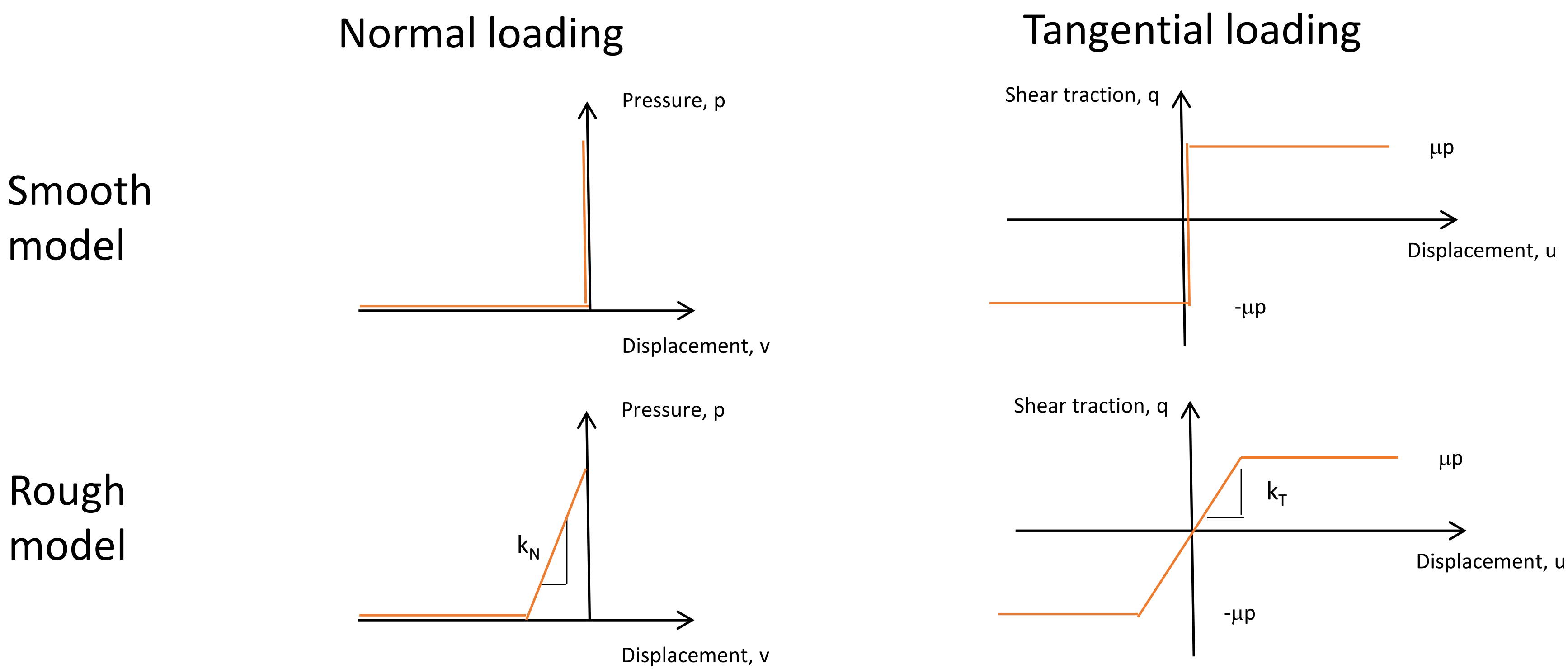
Normal Load Transfer

- Normal load transfer should be straightforward
- Either in contact with a compressive force or out of contact with a tensile force
- However, real surfaces are rough
- Contact only takes place at discrete points
- Therefore there is an additional compliance at the scale of the surface roughness.



Images: Simon Medina

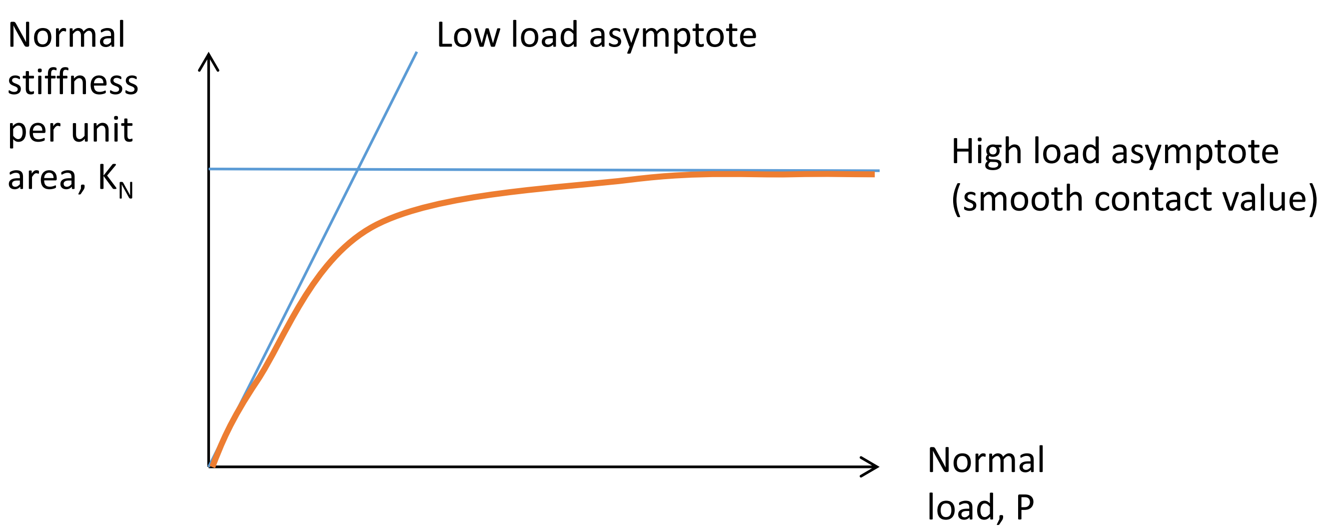
Behaviour of Smooth & Rough Contacts



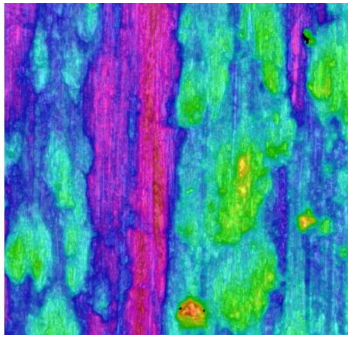
- For a smooth contact model we only need the friction coefficient, μ
  - Predicting μ from surface geometry, material properties, etc. is a difficult problem
- A rough contact model also needs k_N and k_T
  - In principle, these might be rather easier to predict
  - However we need reliable experimental data for model validation

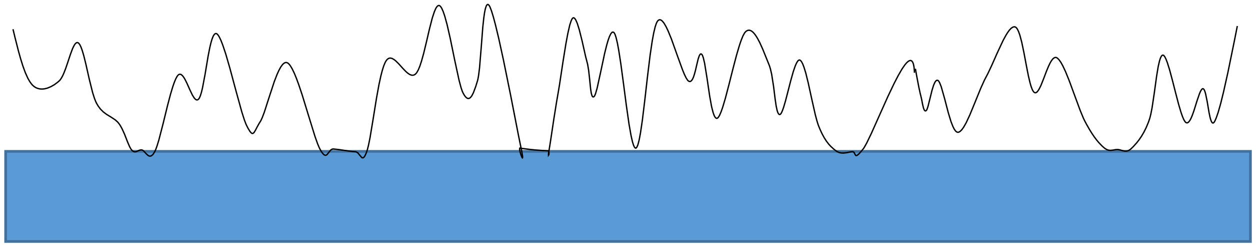
Modelling of rough contact

- A simple model may be formulated along Greenwood-Williamson lines assuming:
  - Hemispherical elastic asperities with constant peak radius
  - Distribution of asperity peak heights is exponential
  - Behaviour of each asperity is independent (i.e. low load)
- Results are given in Medina, Nowell, and Dini (2013):
  - At low loads, normal contact stiffness is proportional to normal load $k_N = \frac{\overline{p}}{\sigma}$
  - At high loads, the stiffness must approach that for an equivalent smooth contact.



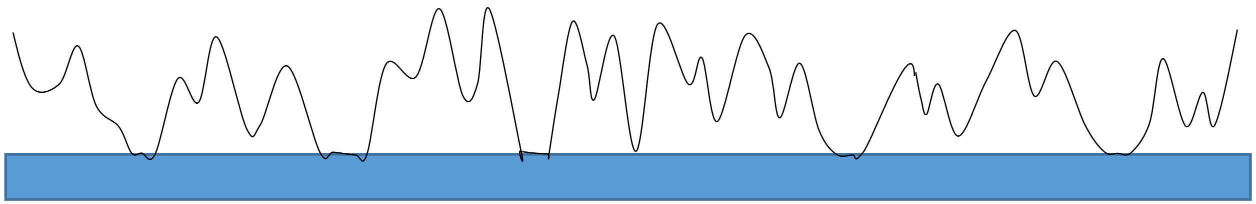
A simple rough contact model





- To develop a model for contact stiffness, we need consider surface roughness
- Initial tangential loading is likely to be predominantly elastic
  - Model will predict tangential stiffness when Q = 0
- Consider a rough elastic surface in contact with a smooth rigid one. This puts all the elasticity and roughness on one surface and is easier to deal with

Formulation



- At light loads, 'asperity' contacts will be relatively widely-spaced and may be modelled as Hertzian

$$p(r) = p_0 \sqrt{1 - \left(\frac{r}{a}\right)^2}$$

- When tangentially loaded, all contacts will initially be 'stuck', so the shear traction at each contact will be given by

$$q(r) = \frac{q_{0i}}{\sqrt{1 - \left(\frac{r}{a_i}\right)^2}}$$

- Mindlin gives the compliance for this traction distribution as

$$\frac{1}{\kappa_i} = \frac{\Delta}{Q_i} = \frac{1}{8a_i}\left(\frac{2-\nu}{G}\right) = \frac{1}{4a_i}\left(\frac{(1+\nu)(2-\nu)}{E}\right)$$

- From this, the Greenwood/Williamson approach can be used to derive an expression for tangential stiffness

Model Result

- The approach leads to

$$\kappa^T = \frac{2(1-\nu)}{(2-\upsilon)}\frac{P}{\sigma}$$

- Note that this is independent of Young's modulus
- This is consistent with the results of Berthoud and Baumberger (1997), who found limited effect of modulus and

$$\kappa = \frac{P}{\lambda}$$

- Where λ is a length scale of the order of microns (i.e. similar to σ)
- Normalisation by area gives

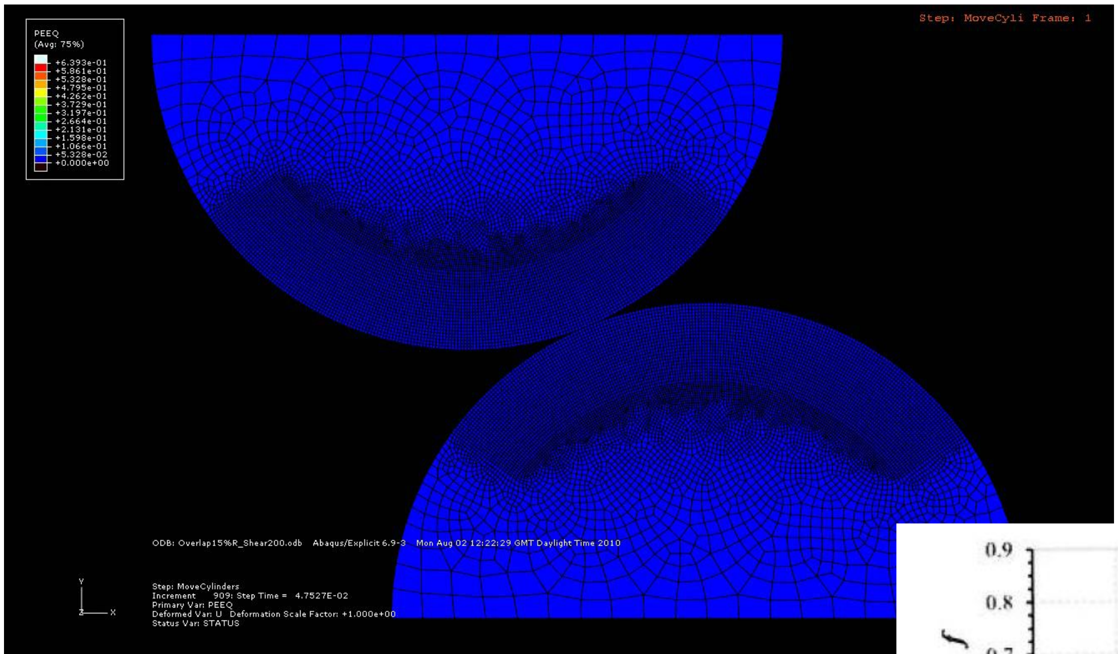
$$\frac{\kappa^T}{A_a} = \frac{2(1-\nu)}{(2-\upsilon)}\frac{\bar{p}}{\sigma} = 0.82\frac{\bar{p}}{\sigma} \quad for\; \upsilon = 0.3$$

- A similar analysis for normal stiffness also predicts stiffness proportional to normal load

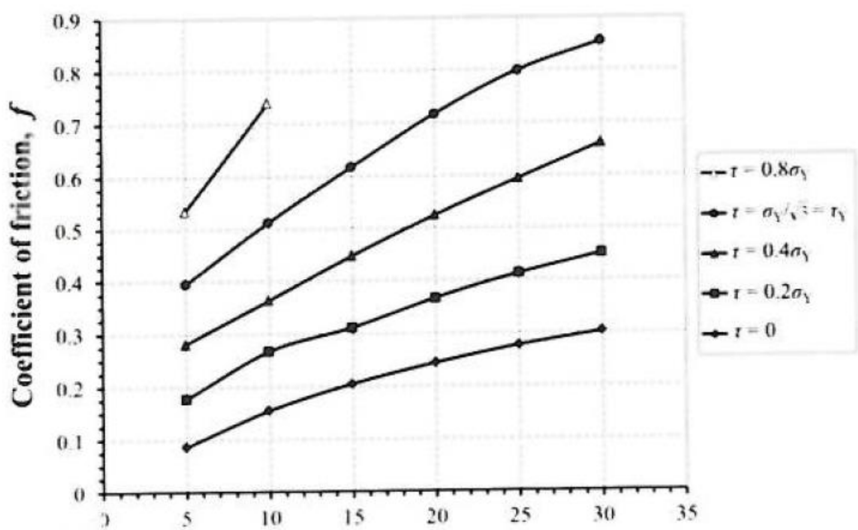
$$\frac{K^N}{A_a} = \frac{\bar{p}}{\sigma}$$

- Hence stiffness is proportional to normal load (pressure) and independent of modulus

Asperity Interaction



FE simulation of asperity interaction can produce realistic friction coefficient values if the overlap and interface strength are high enough.



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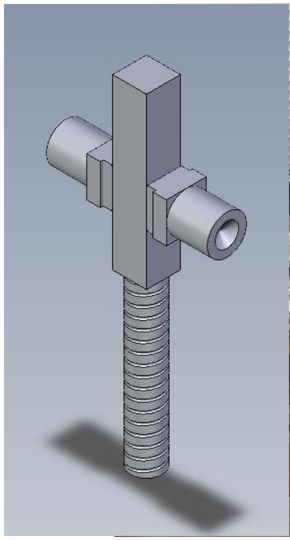
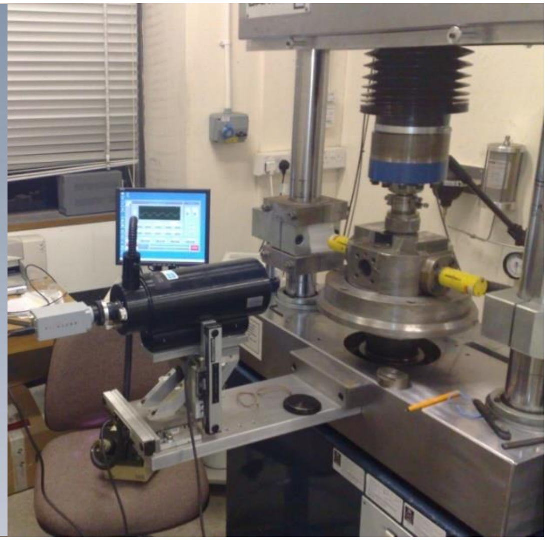
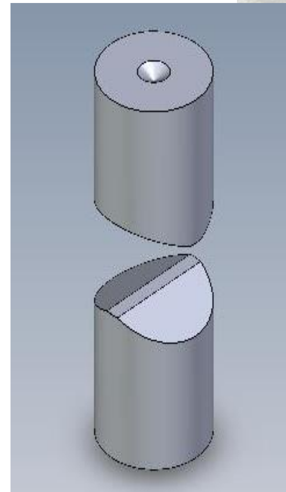
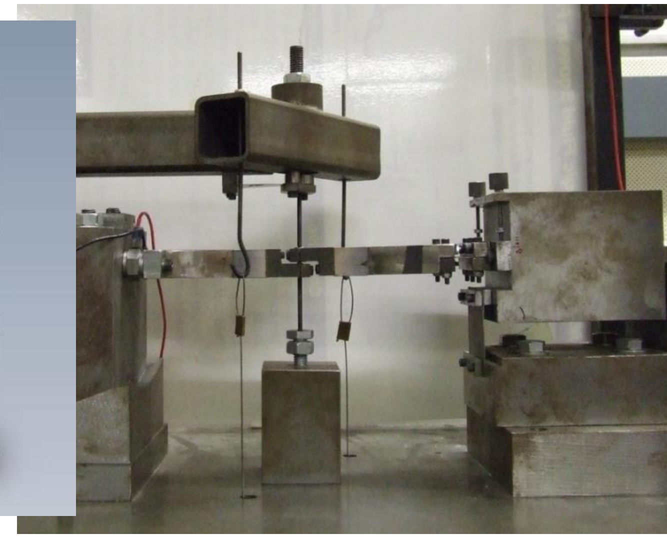
Experimental Techniques

- Two approaches to contact stiffness measurement are available:
  - Direct Measurement of Load and Displacement
    - Digital Image Correlation (Mulvihill, Kartal et al)
    - Laser Velocimetry (Schwingshackl, Gola, et al)
  - Indirect Measurements
    - Ultrasound (Dwyer-Joyce et al)

Oxford and Imperial rigs

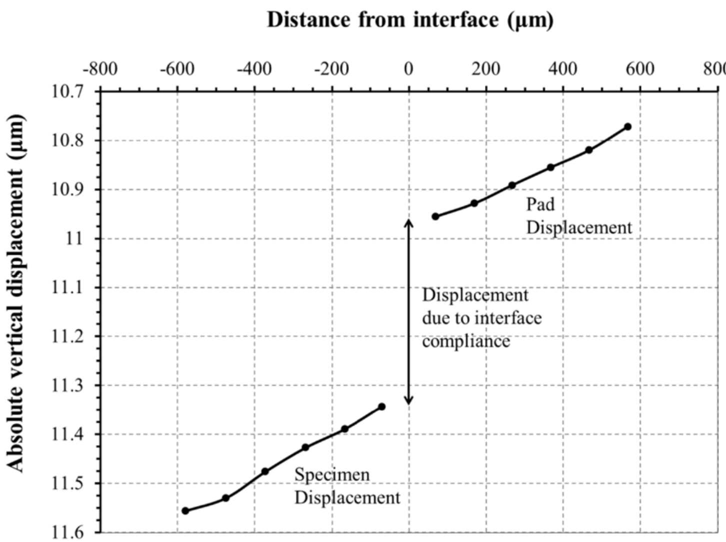
- 80 mm^2 flat and rounded contact
- 1Hz Frequency
- 0.6mm sliding distance
- Displacement measurement by remote LVDT or digital image correlation

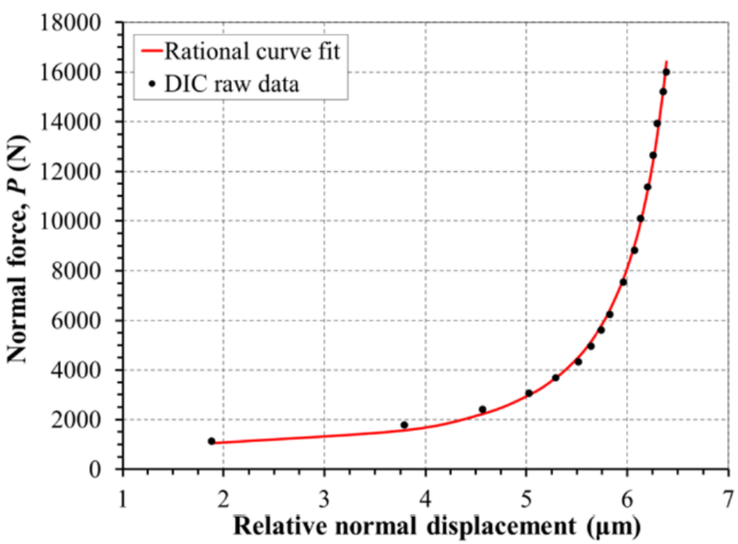



- **1 mm^2 flat on flat contact**
- **~100Hz Frequency**
- **30μm sliding distance**
- **Displacement measurement integration of LDV measurements**

Experimental results

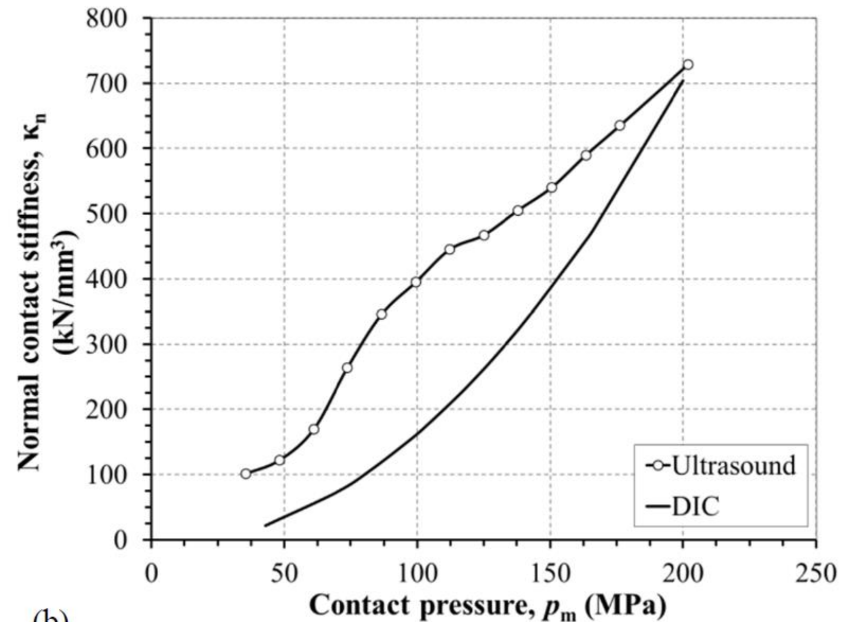
- Digital Image Correlation shows 'gap' in displacement vs position graph
- This can be interpreted as the additional displacement due to the rough contact
- By plotting the variation of this displacement we can obtain a load/displacement relationship
- By fitting a curve and differentiating we can obtain a normal stiffness





Normal stiffness

- Experimental results with Ti 6/4 show stiffness increasing with load
- It is not linear initially, perhaps due to alignment issues
    - We are only looking at the side of the contact
- Reasonable agreement between results and independent measurement using ultrasound
    - Ultrasound samples the entire contact



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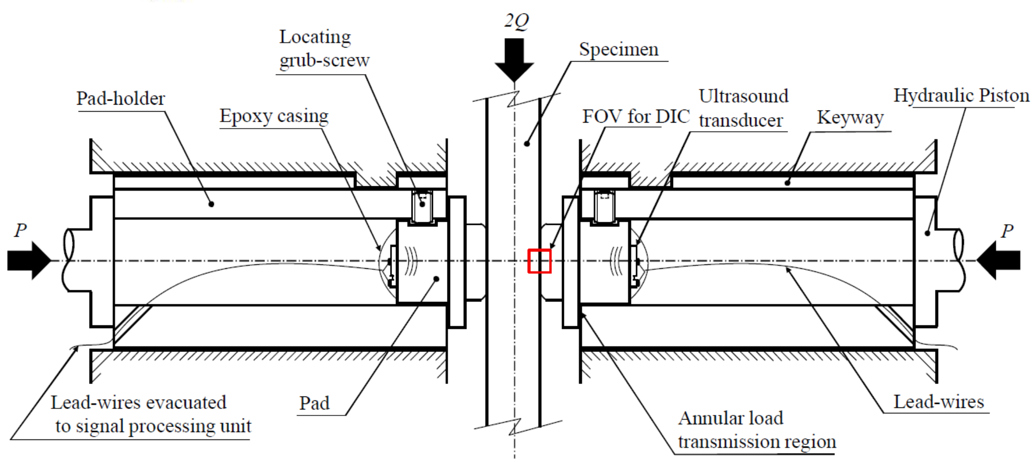
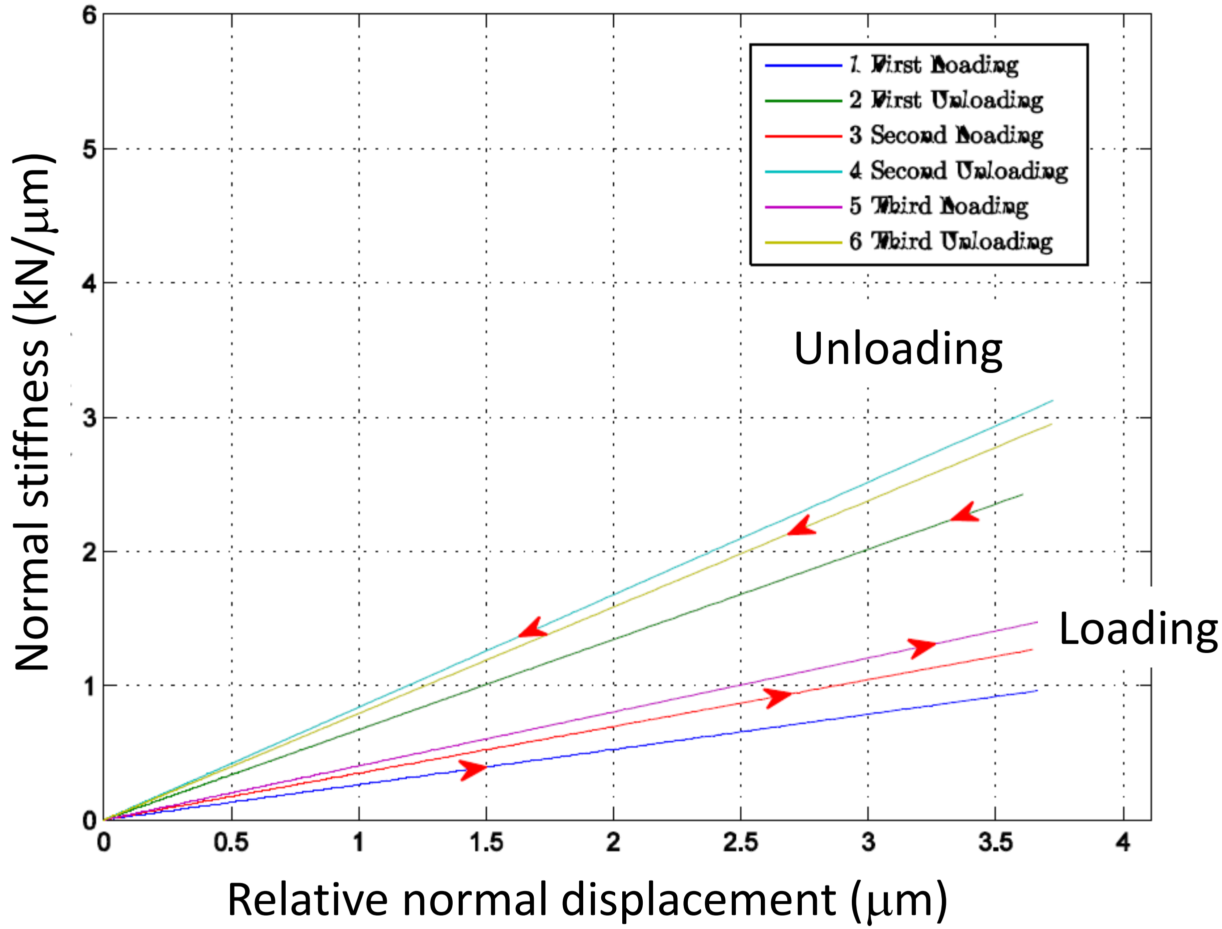
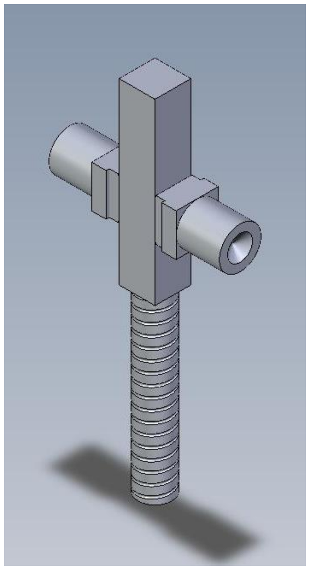


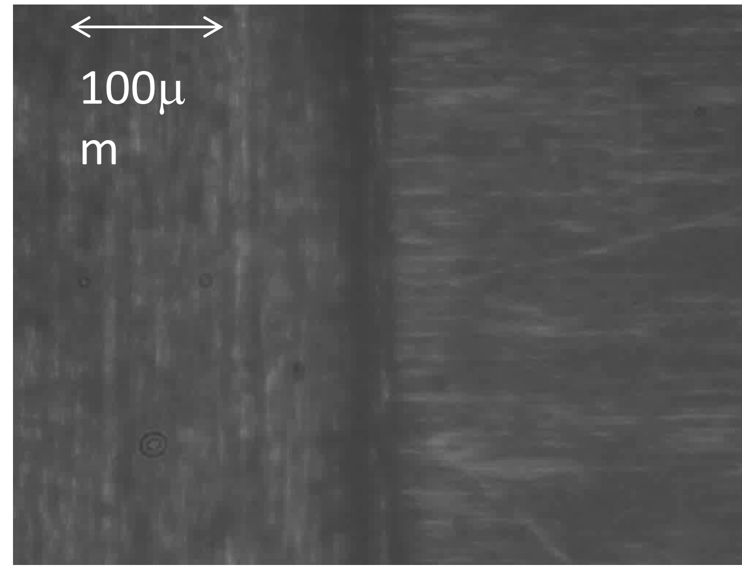
Results: repeated normal loading



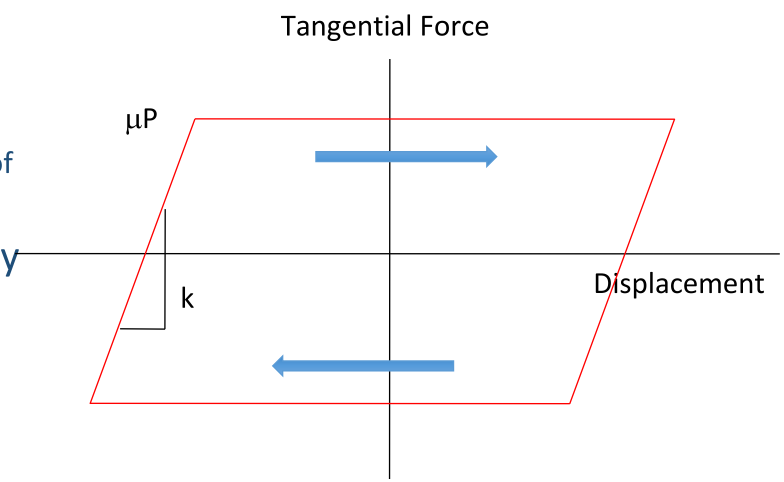
- Unloading stiffness is higher than loading stiffness
- Stiffness increases with each successive loading (hardening behaviour)

Measured and idealised tangential hysteresis loops

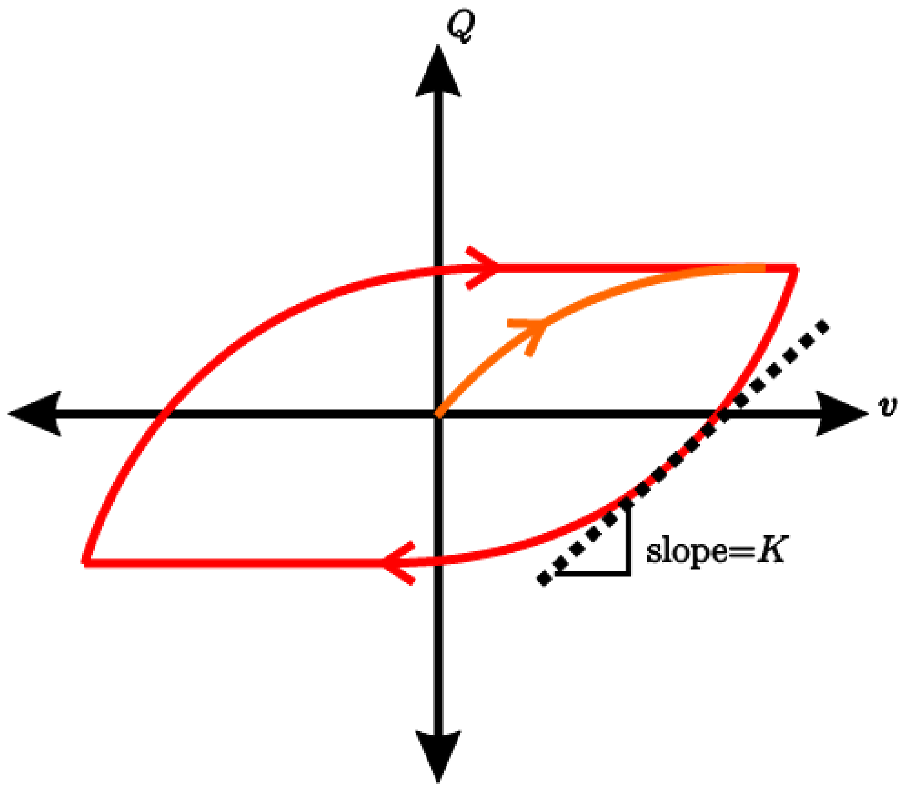


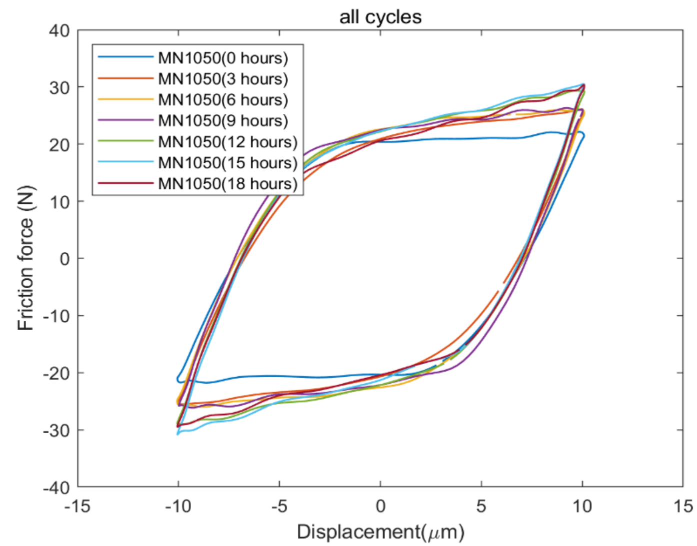


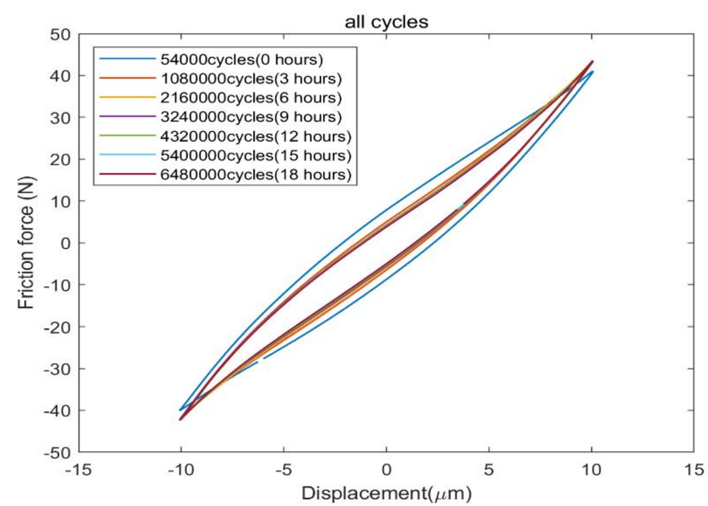
- A bar is clamped between two pads in a test machine
  - Can be loaded in oscillatory sliding and the hysteresis loop measured
  - Digital image correlation can be used to obtain local measurements of displacement
- In tangential loading, the idealised loop is characterised by contact stiffness, k and friction coefficient, μ
  - These can be reasonably representative of real loops (at least initially)
- Similar measurements can be made for normal stiffness
- Ti 6/4 is used in the current work (ground surfaces)



Tangential Stiffness







Tangential stiffness will vary with tangential load as well as with normal load.

Tangential Stiffness

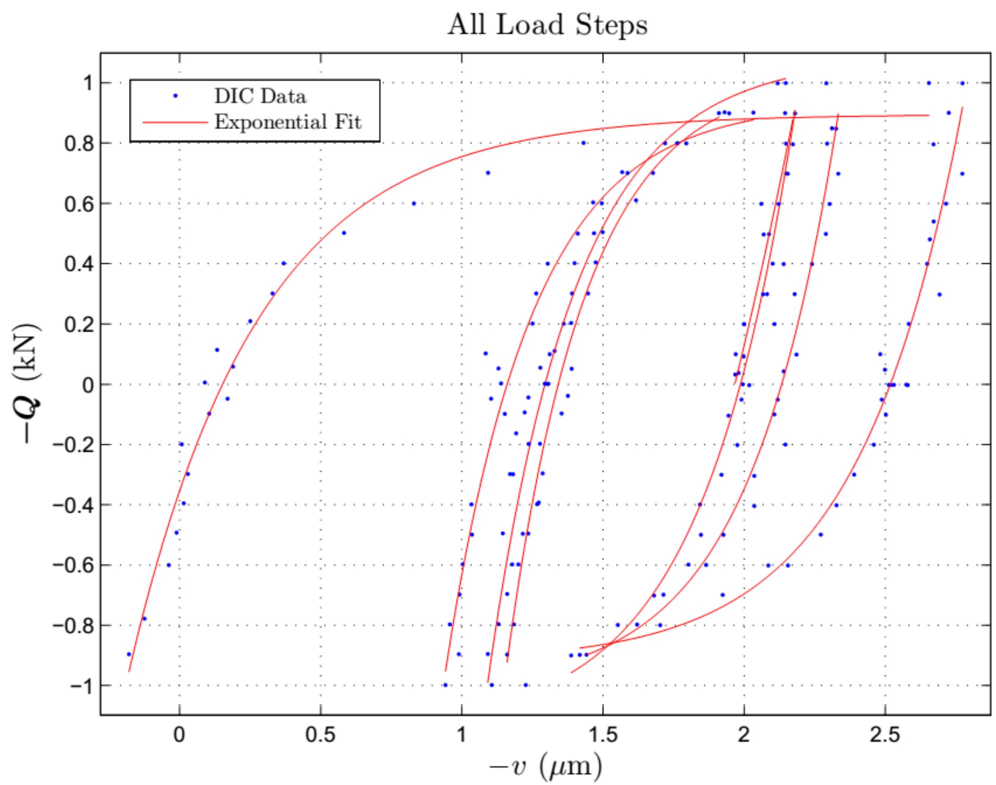
Normal Load (-*P*)= 6 kN
Maximum Shear load= 1 kN

Fit of form
$$-\boldsymbol{Q} = \boldsymbol{A}e^{\boldsymbol{b}(-\boldsymbol{v})} + \boldsymbol{C}.$$

A, *b* and *C* are fit parameters.

The adjusted root mean square between 0.9123 and 0.9732

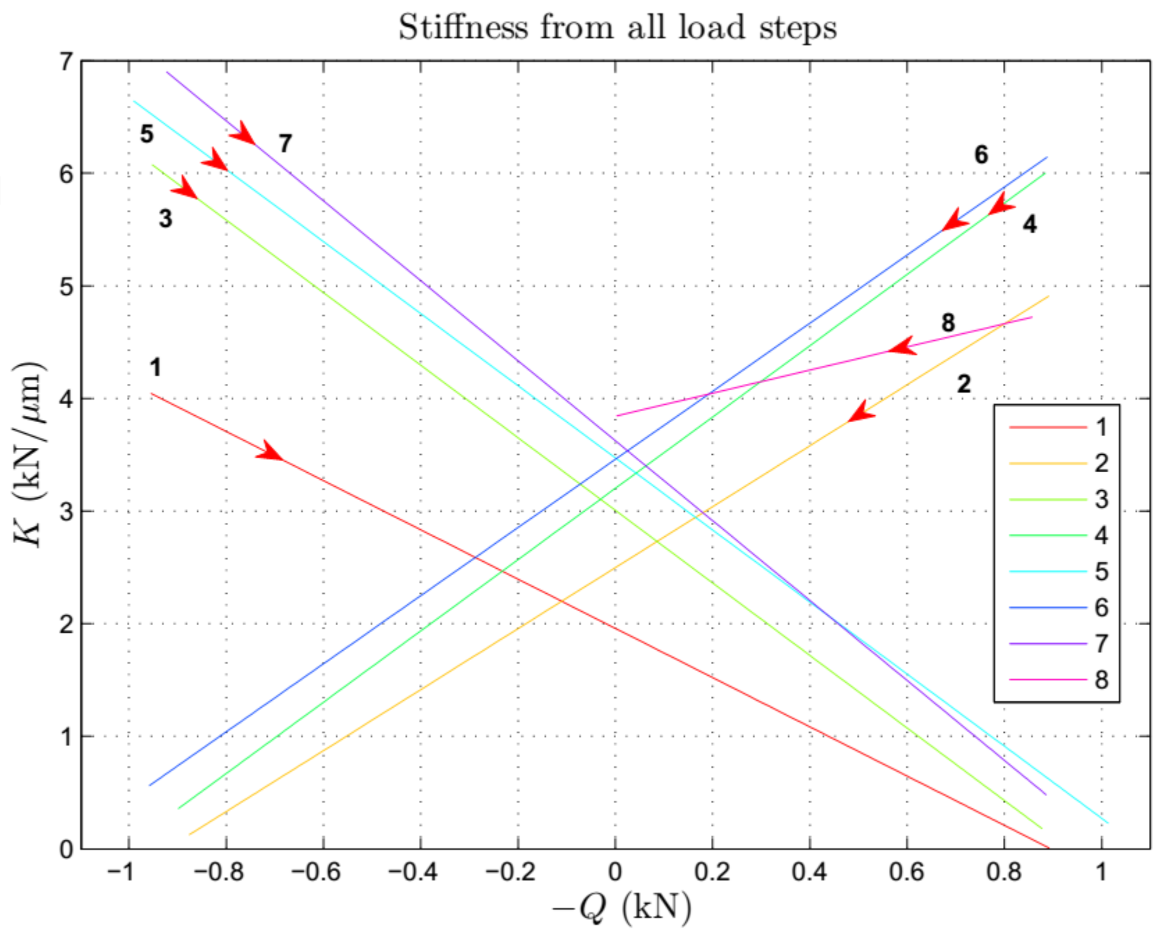
Except for the eight step it is: 0.7502.



Tangential Stiffness

Normal Load (-*P*)= 6 kN
Maximum Shear load= 1 kN

Stiffness derived from the relationship:
$$-\boldsymbol{Q} = \boldsymbol{A}e^{\boldsymbol{b}(-v)} + \boldsymbol{C}.$$



Effect of Normal Load

Tests on same specimen surface with similar pads, for 4kN, 8kN and 12 kN normal loads.

Tangential Stiffness at Q = 0 is seen to approximately double from 4kN to 8kN.

Plot is for the 5th loading and unloading steps

So tangential stiffness also appears proportional to normal load

Effect of Surface Roughness

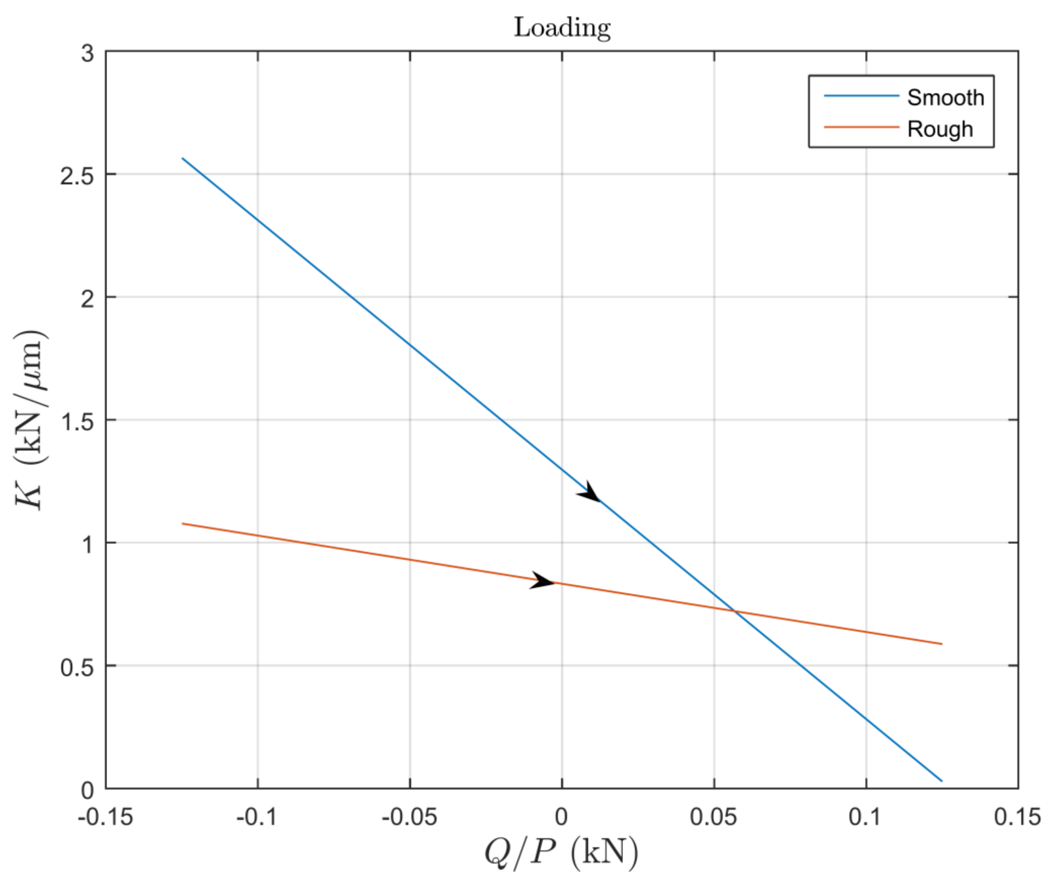
Normal Load (-*P*)= 4 kN
Maximum Shear load=0.5 kN

Smooth Specimen surfaces had roughness (Ra) of 0.568 and 0.351 μm.

The rough specimen surfaces had roughness (Ra) of 1.254 and 1.3051μm.

Pads of similar surface roughness values were used (0.9 to 1.3 μm)

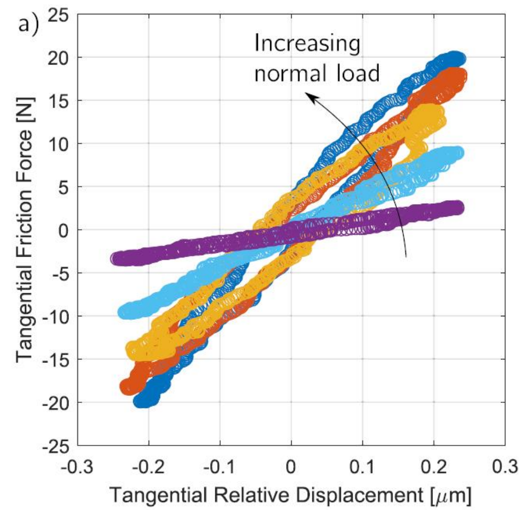
Plot is for 5th loading step.

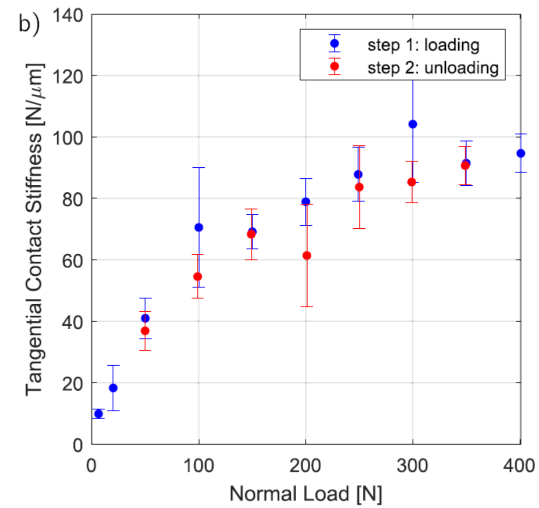


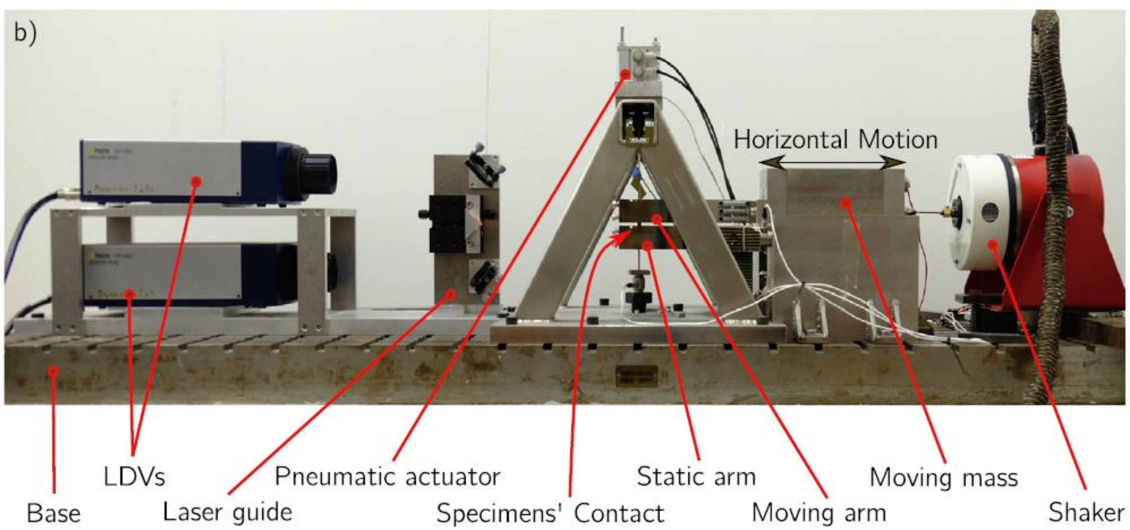
Measurements during vibration

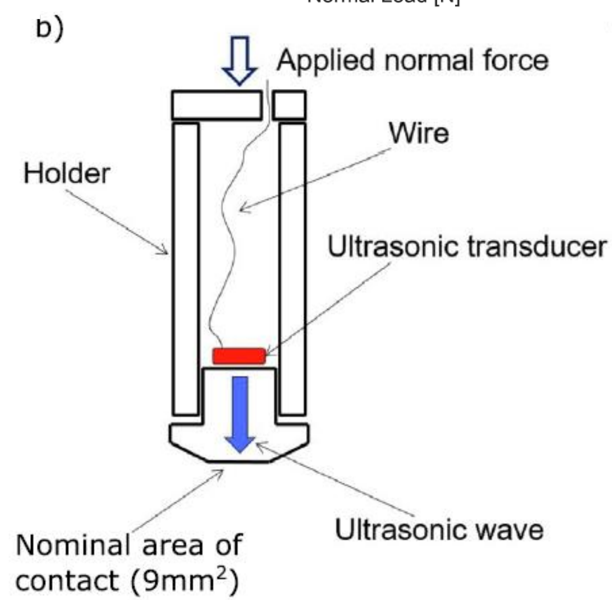
Work shown so far (Kurien Parel and Daniel Mulvihill) is essentially quasi-static

Recently Alfredo Fantetti has extended this to ultrasound measurements made during the course of a vibration experiment



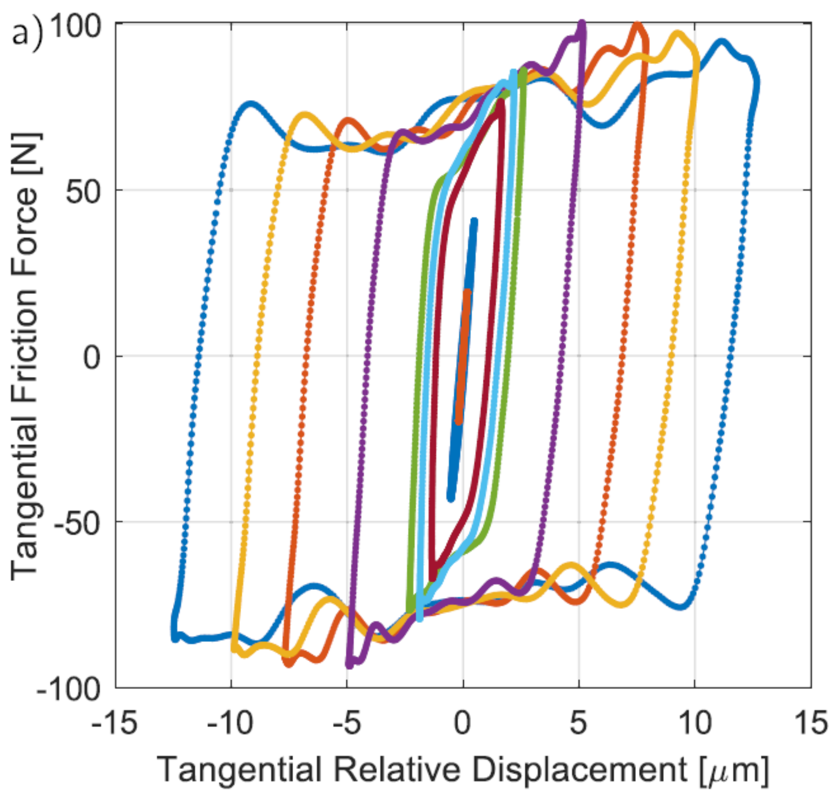


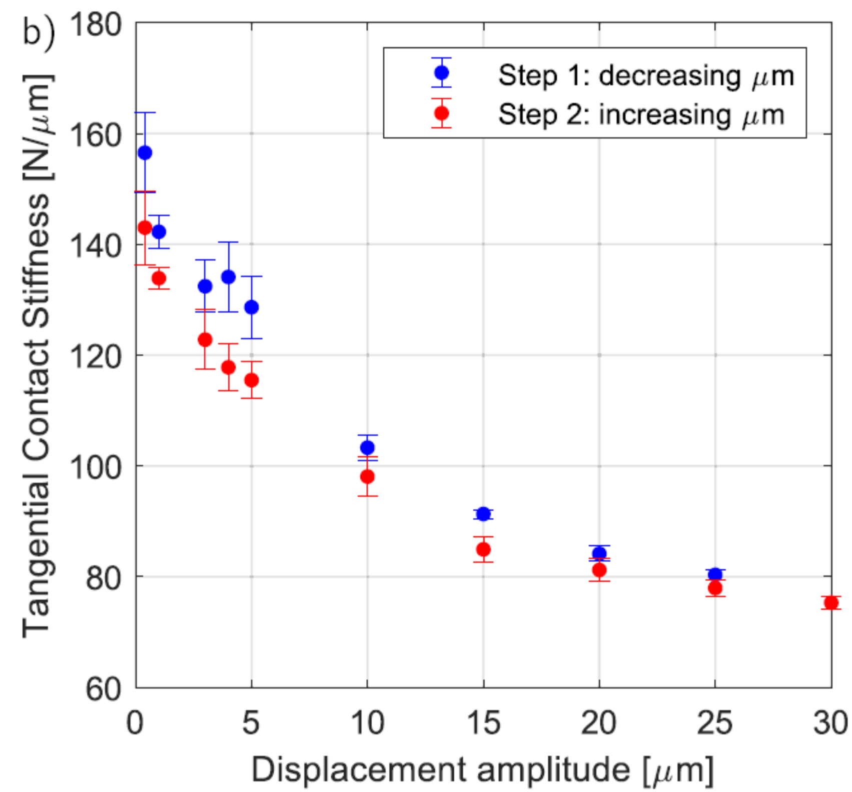




Measurements during reciprocating sliding

Measurements made at different amplitudes of reciprocating sliding show that the contact stiffness recorded on load reversal decreases as the applied displacement is increased and the contact moves into the sliding regime

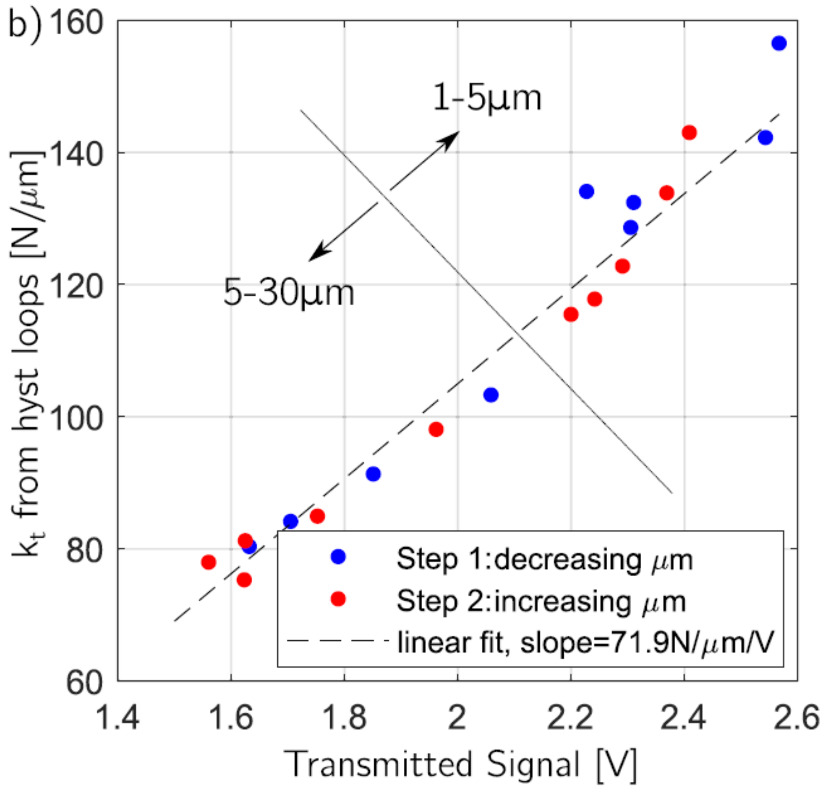


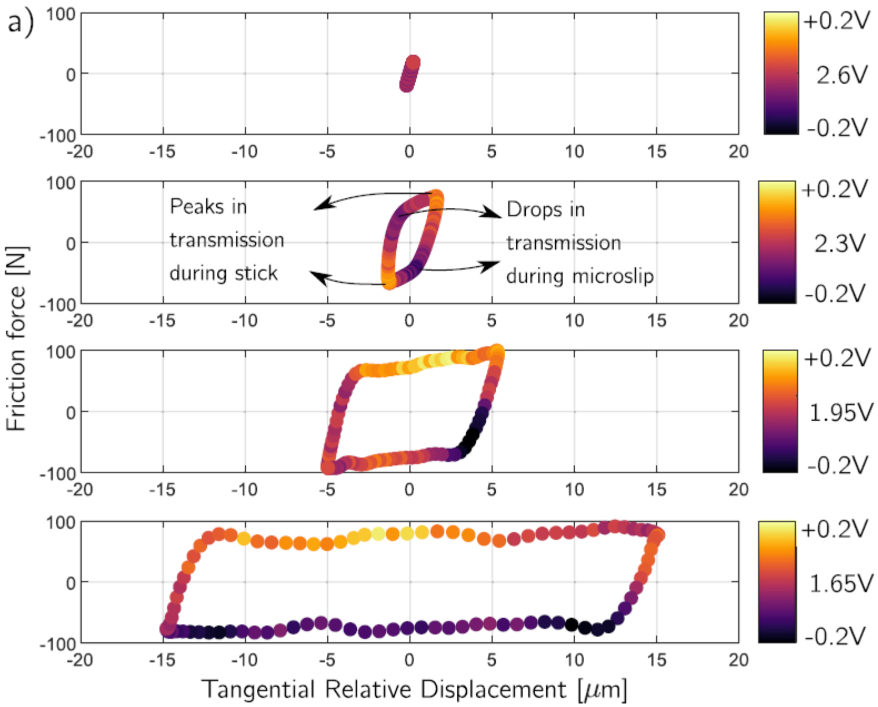


Measurements during reciprocating sliding (2)

Good correlation is found between the tangential stiffness obtained from ultrasound with that obtained from LDV measurements. However, it is difficult to compare numerical values, as the area sampled is different.

Maximum stiffness was recorded in the stick phase of contact interface. Slight asymmetry was detected from a change in normal load due to apparatus design.





Conclusions – the way forward?

- Stiffness measurements are not straightforward
  - However, we are starting to develop reliable and independent measurement techniques
- Stiffness is not a surface property
  - Normal stiffness will depend on normal pressure
  - Tangential stiffness will depend on normal pressure and on local shear traction
  - Stiffness may increase with repeated loading
- Friction will be much more difficult to predict than stiffness
  - However, we do need physics-based models of these phenomena
- There remains the challenge of how to incorporate such complexity in our joint models – how much do we need?
- Can we use our understanding to design:
  - More repeatable joints
  - Joints with specific properties
- The tribology community needs to understand more about vibrations and dynamicists need to understand more about tribology – Hence Tribomechadynamics

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