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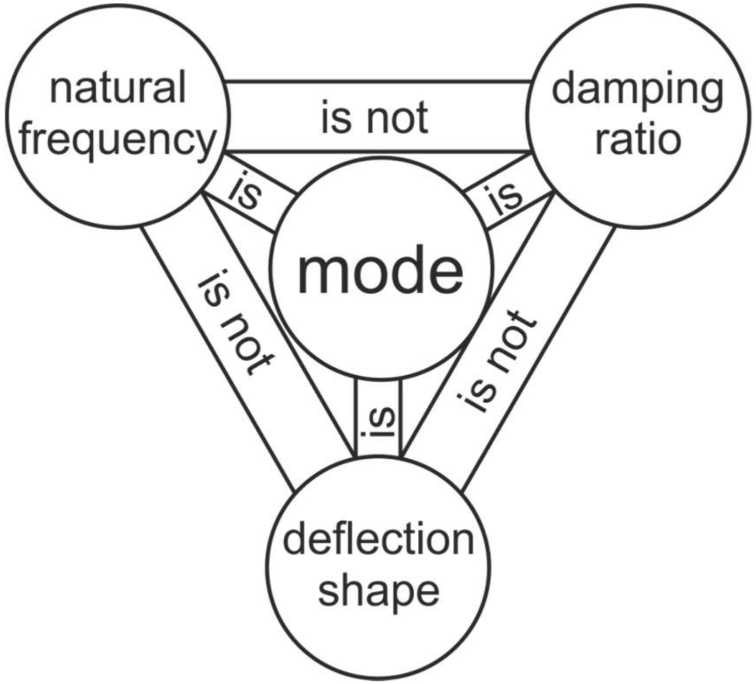
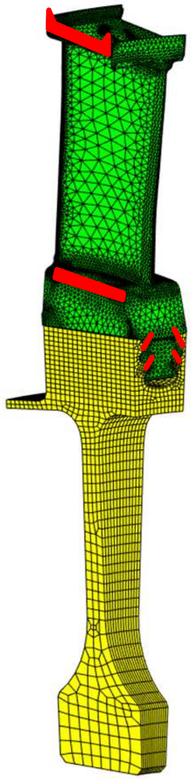
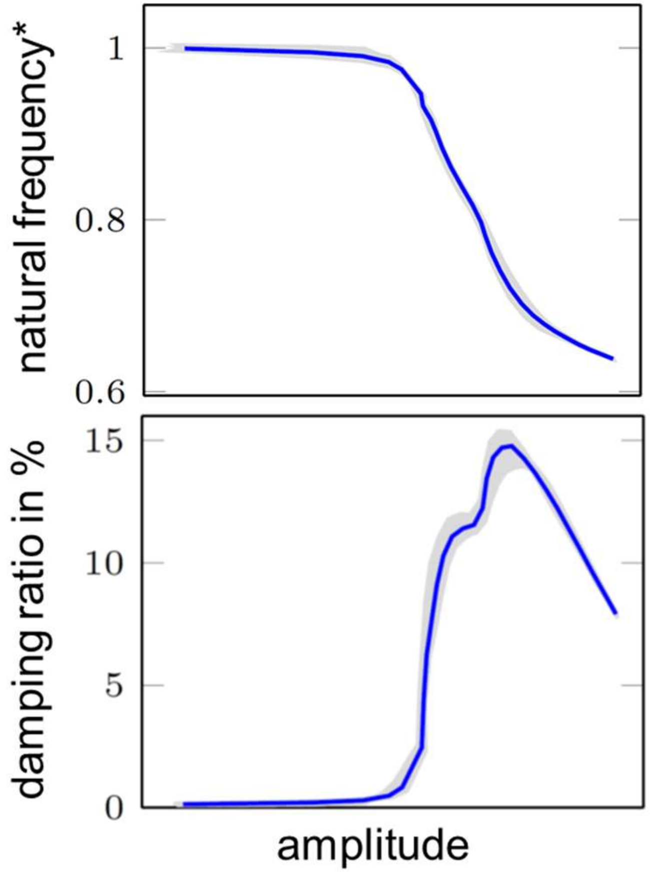
Nonlinear Modal Testing of Jointed Structures

Prof. Dr.-Ing. Malte Krack







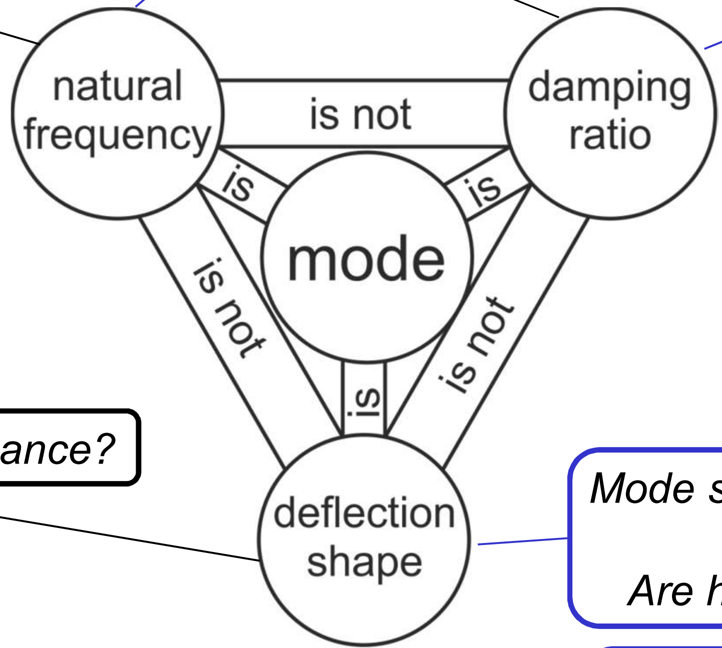
Modes characterize the vibration signature.

Positive damping or self-excitation?
Magnification at resonance?
Quick decay of free vibrations?

Hardening or softening?
Resonance frequency shift?

Time scale of vibration?
Where do resonances occur?

In- or decreasing with amplitude?
Overall damping capacity?



modal trinity

Spatial distribution at resonance?

Mode shape change? Localization?
Modal interactions?
Are higher harmonics relevant?

linear
modal properties <u>amplitude-independent</u>

nonlinear
modal properties <u>amplitude-dependent</u>

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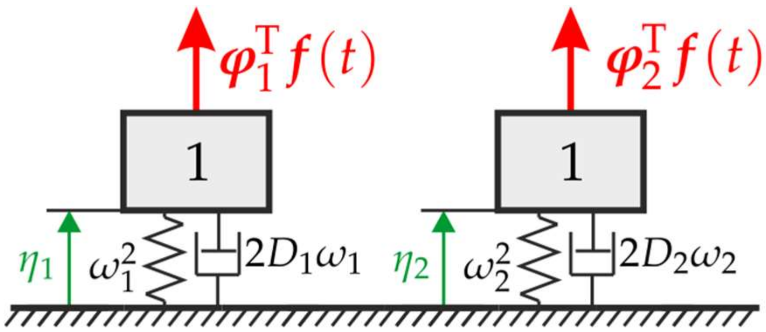
Modes simplify the quantitative analysis of the vibration response.

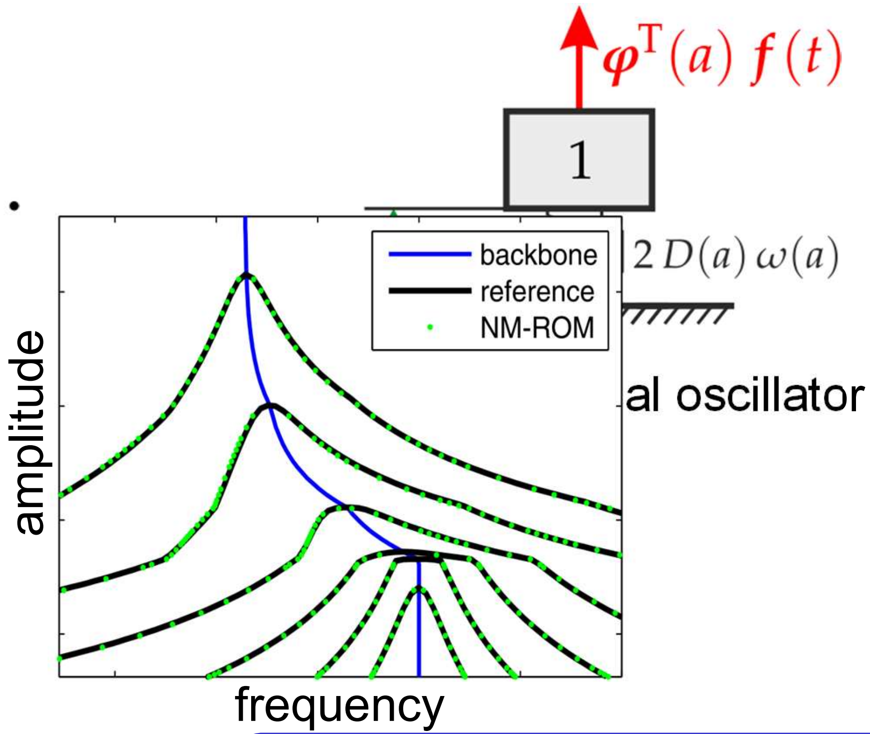
Superposition & Orthogonality



decoupled modal oscillators

. . .

Single-Nonlinear-Mode Theory



linear
modal properties <u>amplitude-independent</u>

nonlinear
modal properties <u>amplitude-dependent</u>

Outline

Introduction

Methodology

- Definition of Nonlinear Modes
- Single-Nonlinear-Mode Theory
- Nonlinear Modal Testing

Results

- selected examples
- lessons learned

Conclusions & Outlook

Extended Periodic Motion Concept (EPMC)[1]

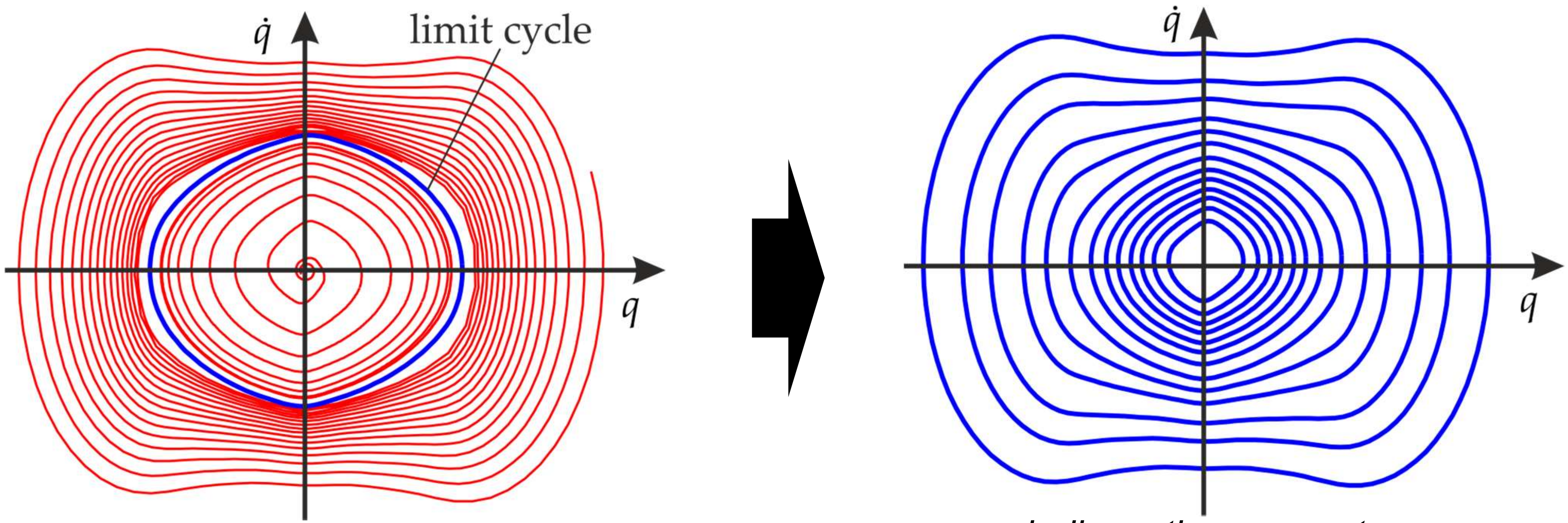
Nonlinear Mode:

- periodic motion of autonomous system
- continuously extends a corresponding linear mode from the equilibrium position

$$\boldsymbol{M}\ddot{\boldsymbol{q}} - 2D\omega\boldsymbol{M}\dot{\boldsymbol{q}} + \boldsymbol{g}\left(\boldsymbol{q}, \dot{\boldsymbol{q}}\right) = \boldsymbol{0}$$

artificial negative damping term compensates natural dissipation

- ➢ consistent with conservative case and with linear case under modal damping
- ➢ If damping is not light and, at the same time, multiple linear modes participate, the artificial term may distort the modal coupling.



damped motion concept *periodic motion concept*

[1] Krack, Computers & Structures, 2015
[2] Jahn et al., MSSP, 2019

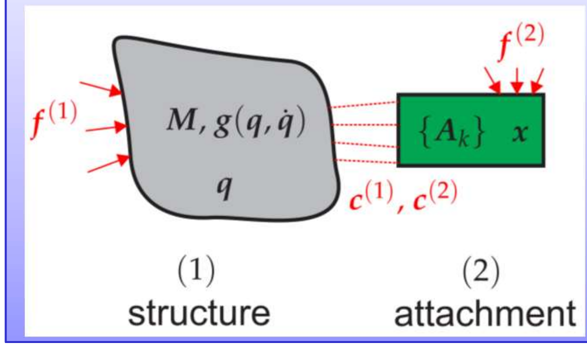
- ➢ rigorous treatment of damping
- ➢ better represents periodic behavior than damped motion concept[2]
- ➢ useful for development of computational and experimental methods

In important cases, a single Nonlinear Mode dominates the response:
well-separated primary resonance; self excitation by negative damping; …

Equation of motion
with *additional terms*

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{g}\left(\boldsymbol{q},\dot{\boldsymbol{q}}\right) = \boldsymbol{f}$$

- $-\boldsymbol{D}\dot{\boldsymbol{q}}$ light damping
- $\boldsymbol{f}_{\mathrm{ex}}(t)$ imposed forcing
- $\boldsymbol{c}$ coupling forces to linear attachments*



*sub-systems described by linear time-invariant ODEs/DAEs[3]

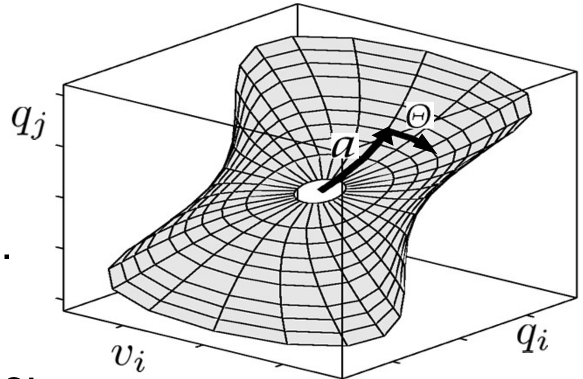
Examples: electric, magnetic, aerodynamic models

Applications: model updating, structural modification, energy harvesting, smart structures, controller design, …

[3] Krack, JSV, 2021

Reduction to the Nonlinear Mode's invariant manifold[4]:

$$\boldsymbol{q}\left(a,\theta\right) = \Re\left\{a\sum_h \boldsymbol{\varphi}_h \mathrm{e}^{\mathrm{i}h\tau}\right\} \qquad \tau = \theta + \int_0^t \Omega(\bar{t})\mathrm{d}\bar{t}$$



Parameters and new variables a, θ allowed to vary slowly with time.

Nonlinear projection onto fundamental harmonic (imprecise if higher harmonics important, but) yields closed-form expression of nonlinear terms:

$$2\mathrm{i}\Omega\left(\dot{a} + \mathrm{i}a\dot{\theta}\right) + \left(\ -\Omega^2 + 2D\left(a\right)\omega\left(a\right)\mathrm{i}\Omega + \omega^2\left(a\right)\ \right)a = \boldsymbol{\varphi}^{\mathrm{H}}\left(a\right)\ \hat{\boldsymbol{f}}\mathrm{e}^{-\mathrm{i}\theta}$$

[4] Krack et al., MSSP, 2014

The reduction to a single Nonlinear Mode simplifies the analysis a lot.

Slow flow equations (= non-parametric, data-driven model):

$$2\mathrm{i}\Omega\left(\dot{a}+\mathrm{i}a\dot{\theta}\right)+\left(\ -\Omega^2+2D\left(a\right)\omega\left(a\right)\mathrm{i}\Omega+\omega^2\left(a\right)\ \right)a=\boldsymbol{\varphi}^{\mathrm{H}}\left(a\right)\ \hat{\boldsymbol{f}}\mathrm{e}^{-\mathrm{i}\theta}$$

→ Splitting into real and imaginary part yields system of two explicit first-order ODEs.

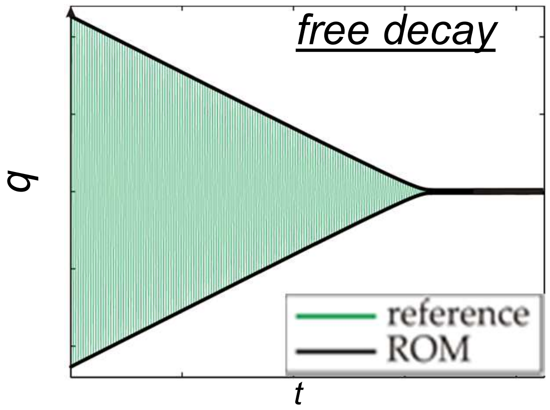
→ Classical analysis of fixed points and their bifurcations.

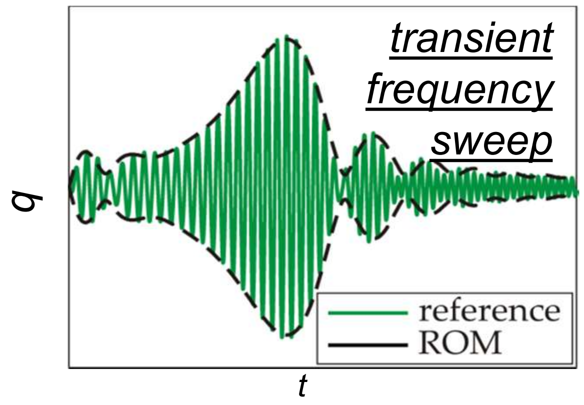
→ Elimination of phase yields scalar algebraic equation for amplitude of fixed points.

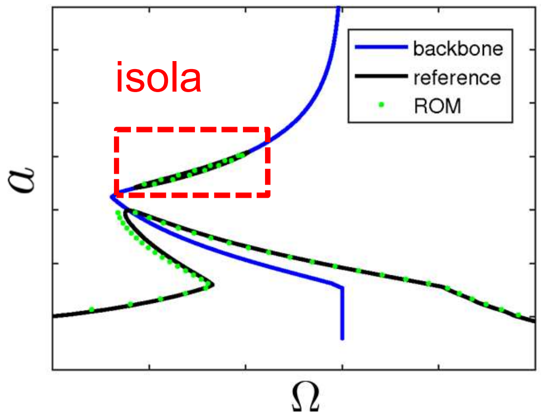
→ Makes parameter studies, probabilistic analyses and design optimization feasible.

→ Closed-form expression of frequency response (also useful to analyze isola)[10]:

$$\left(\frac{\Omega}{\omega}\right)^2=1-2D^2\pm\sqrt{\left(\frac{\left|\boldsymbol{\varphi}^{\mathrm{H}}\hat{\boldsymbol{f}}\right|}{a\omega^2}\right)^2-4D^2\left(1-D^2\right)}$$







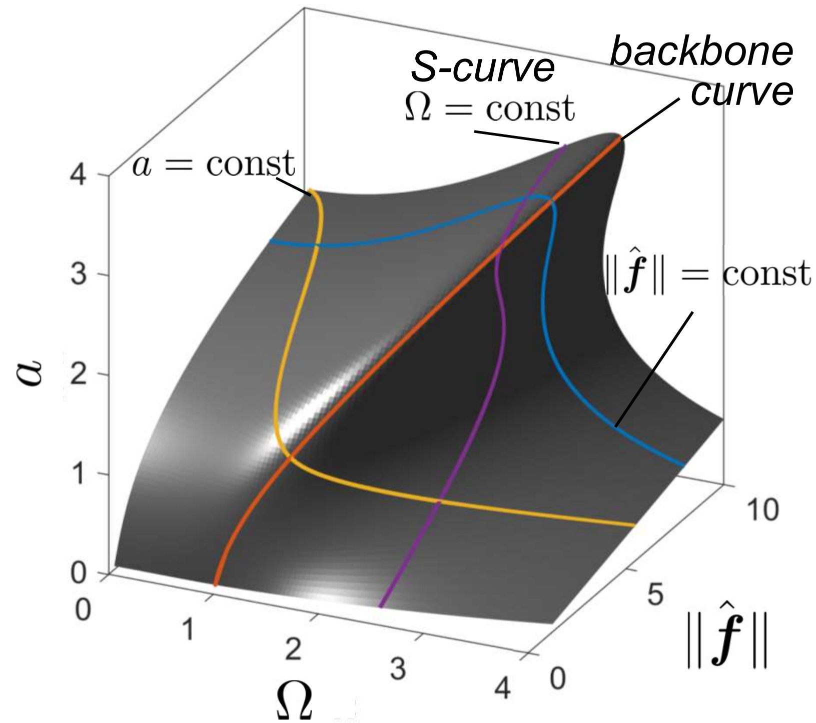
[10] Schwarz et al., GTP, 2019

Standard methods for computation of Nonlinear Modes are available, while experimental methods are still under research.

Usefulness: model validation and updating;
system identification (e.g. for control; non-parametric → no prior knowledge of form and location of nonlinearities needed)

Methods

- Impact Hammer Testing[5]
  → limited to weak rather nonlinearity
- Shaker Testing
  – whole frequency response surface
    - outer loop: amplitude, inner loop: frequency[6,7]
      → quasi-linear; state-of-technology
    - outer loop: frequency, inner loop: amplitude[8]
  – only backbone curve (or close to it[9])
    - manual vs. feedback-controlled phase
    - identification from free decay vs. steady state



frequency response surface

➢ less test data needed
➢ high robustness
➢ simple signal processing

[5] Deaner et al., JVA, 2015
[6] Link et al., IMAC, 2011
[7] Özgüven, MSSP, 2021
[8] Renson et al., JSV, 2016
[9] Kwarta&Allen, MSSP, 2022
[16] Peeters et al., JSV, 2011

Nonlinear Modal Analysis via phase-resonant testing[11]

linear Modal Analysis → tracking of phase-resonance curve in specified amplitude range → identification of amplitude-dependent modal properties

$$\omega(a), D(a), \boldsymbol{\varphi}(a)$$

How to identify nonlinear modal properties (inverse problem)?

$$2\mathrm{i}\Omega\cancel{\left(\dot{a}+\mathrm{i}a\dot{\theta}\right)}+\left(\;-\Omega^2+2D\left(a\right)\omega\left(a\right)\mathrm{i}\Omega+\omega^2\left(a\right)\;\right)a=\underbrace{\boldsymbol{\varphi}^{\mathrm{H}}\left(a\right)\;\hat{\boldsymbol{f}}\mathrm{e}^{-\mathrm{i}\theta}}$$

steady state

phase resonance: $=\mathrm{i}\|\hat{f}\|$

Frequency is output of PLL $\omega=\Omega$

Damping results from power balance

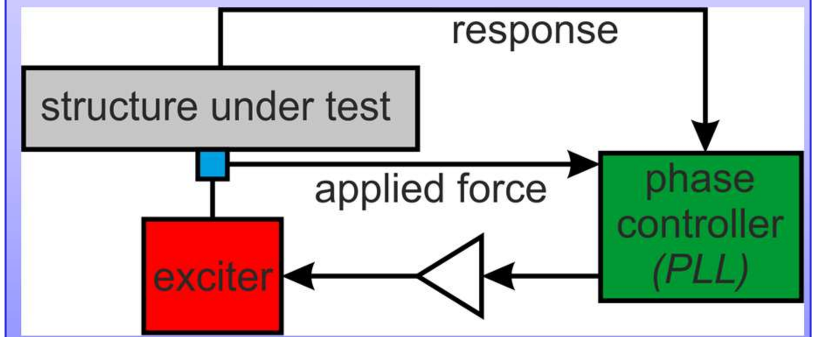
$$D=\frac{\left|\hat{\boldsymbol{q}}_1^{\mathrm{H}}\hat{\boldsymbol{f}}\right|}{2\omega^2a^2}=\frac{P_1}{\omega^3a^2}$$

Modal mass is estimated using linear mass-normalized mode shapes

$$a=\sqrt{\hat{\boldsymbol{q}}_1^{\mathrm{H}}\;\boldsymbol{M}\;\hat{\boldsymbol{q}}_1}=\|\boldsymbol{\Phi}_{\mathrm{lin}}^{+}\;\hat{\boldsymbol{q}}_1\|$$

$$\hat{\boldsymbol{q}}_1=a\boldsymbol{\varphi}(a)$$

example of implementation using single-point forcing



[11] Scheel et al., JSV, 2018

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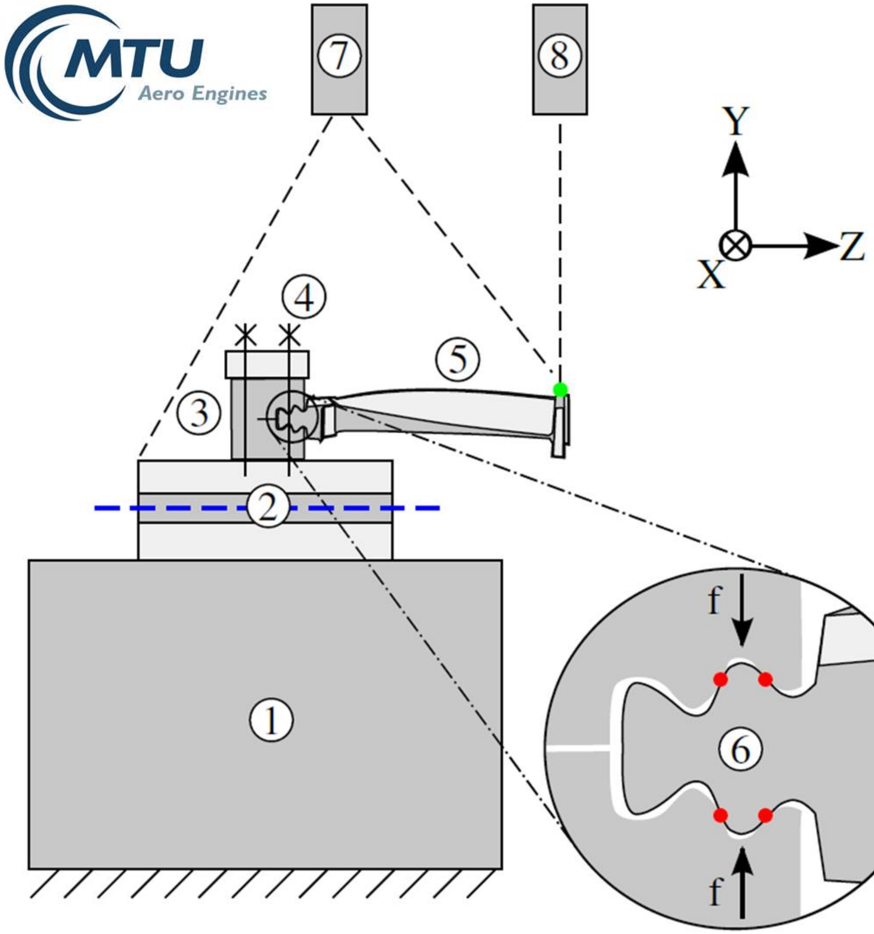
The proposed method has been validated for a series of test rigs.

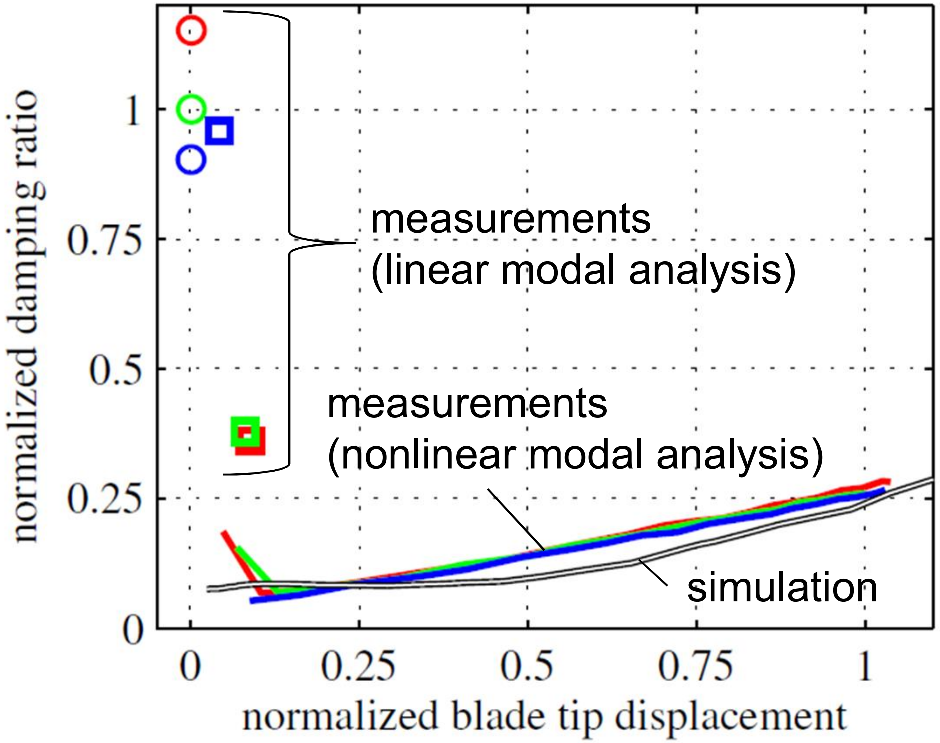
Test rig	geometric NL	dry friction	multi-physical NL
ECL benchmark	✓		
Joint resonator		✓	
Beam with repelling magnets			✓
BRB		✓	
Engine blade with fir-tree joint	✓	✓	
RubBeR		✓	
Clamped-clamped beam	✓	✓	

Validation
The predictions using Single-Nonlinear-Mode Theory are almost always within the repeatability spread of the reference measurements.

➢ **Method quantifies modal properties with high accuracy, including damping!**

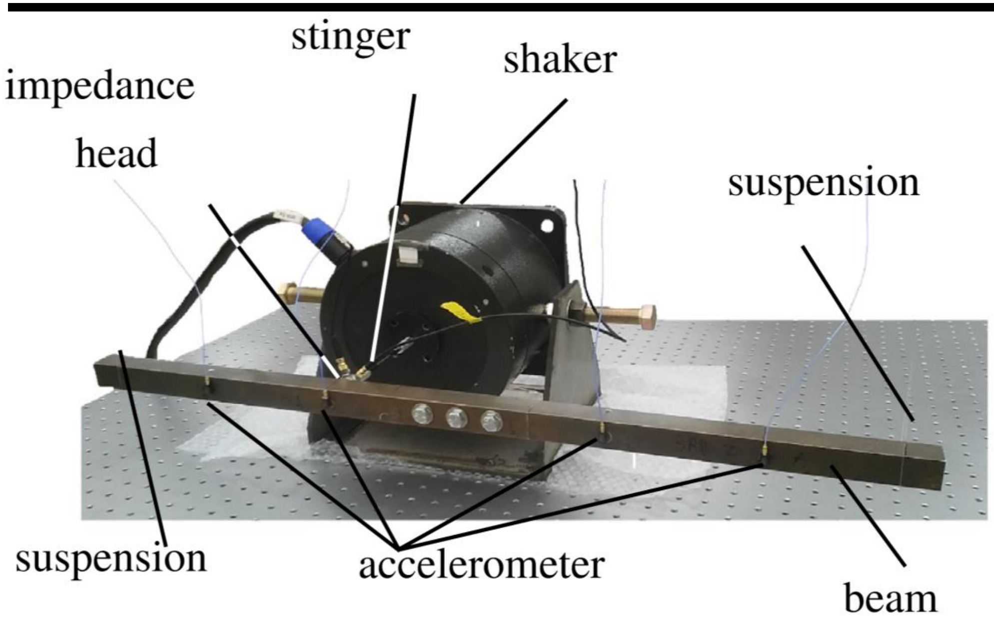
Results: Engine blade with fir-tree joints

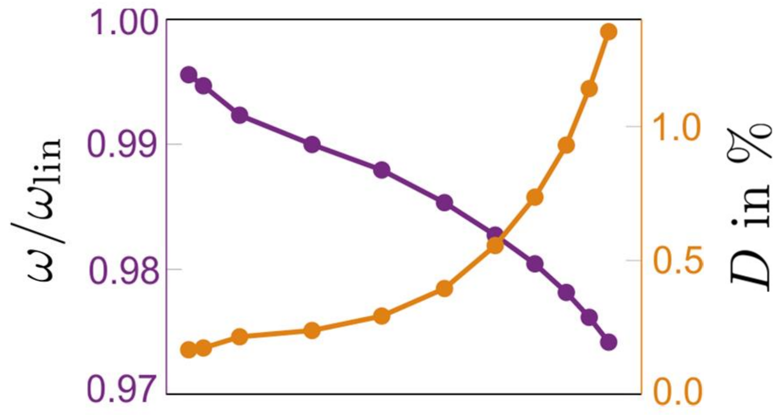


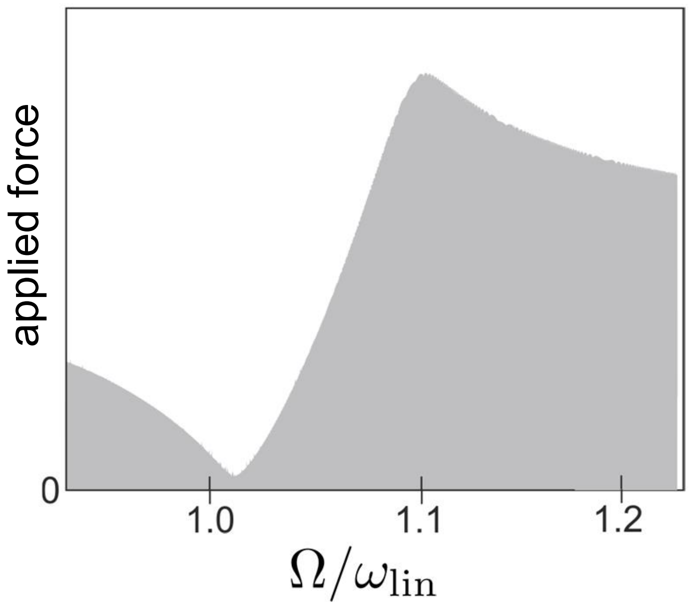


- Successful implementation in industrial environment demonstrates the method's maturity.
- Method is useful for validation and assessment of simulations.
- Analyses reveal limitations of contact model w.r.t. friction damping in low-amplitude regime.

[10] Schwarz et al., GTP, 2019







- Typical force drop during frequency sweeps through resonance (without control).
- Single-Nonlinear-Mode Theory produces large errors when the measured force is imposed.

[12] Scheel et al., MSSP, 2020

Results: Brake-Reuss-Beam

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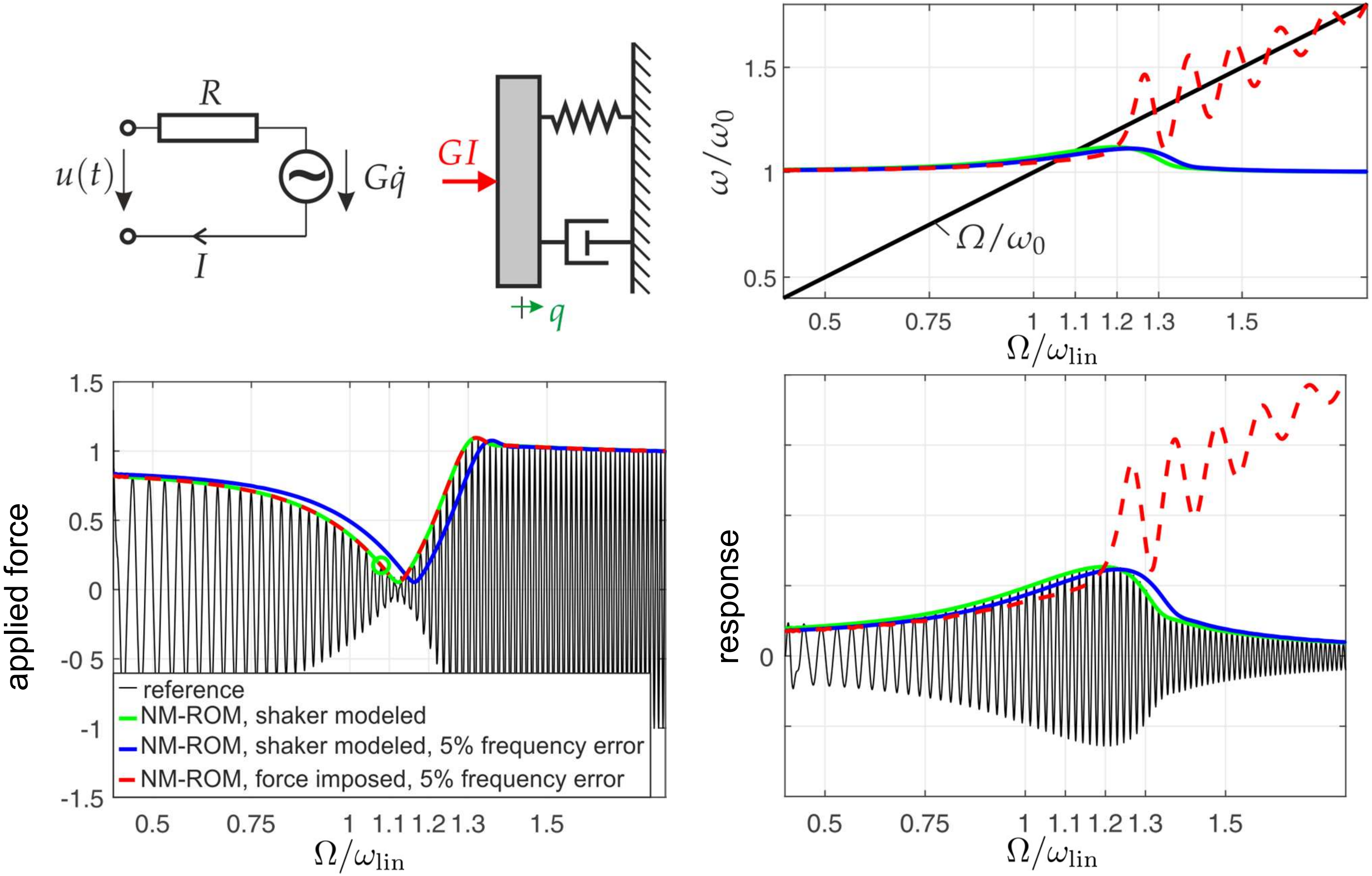

- ➢ The electro-mechanical model of the shaker must be included as linear attachment.
- ➢ Ignoring the physical cause of the force drop induces high sensitivity to frequency errors.
- ➢ Actually neither a transient nor a nonlinear phenomenon.

[3] Krack, JSV, 2021

Results: Rubbing Beam Resonator[13]

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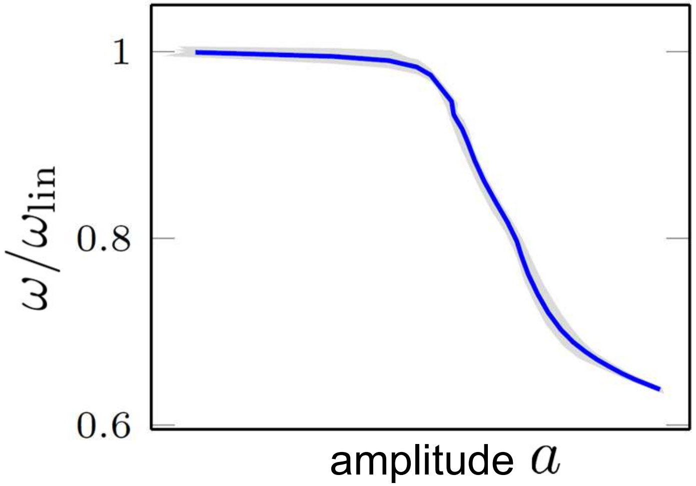

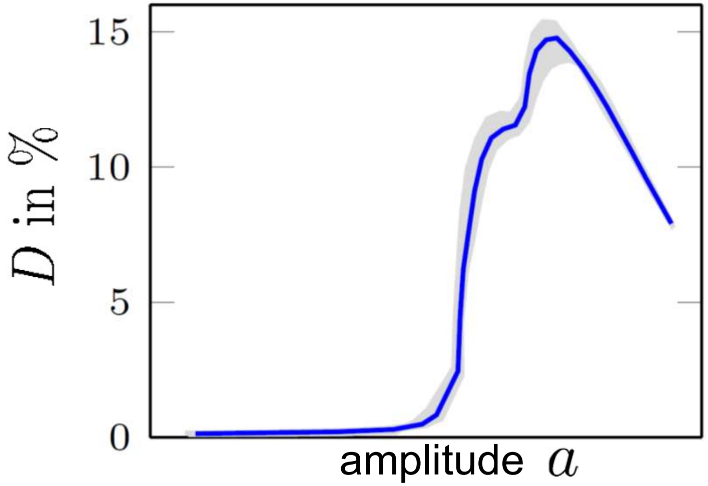

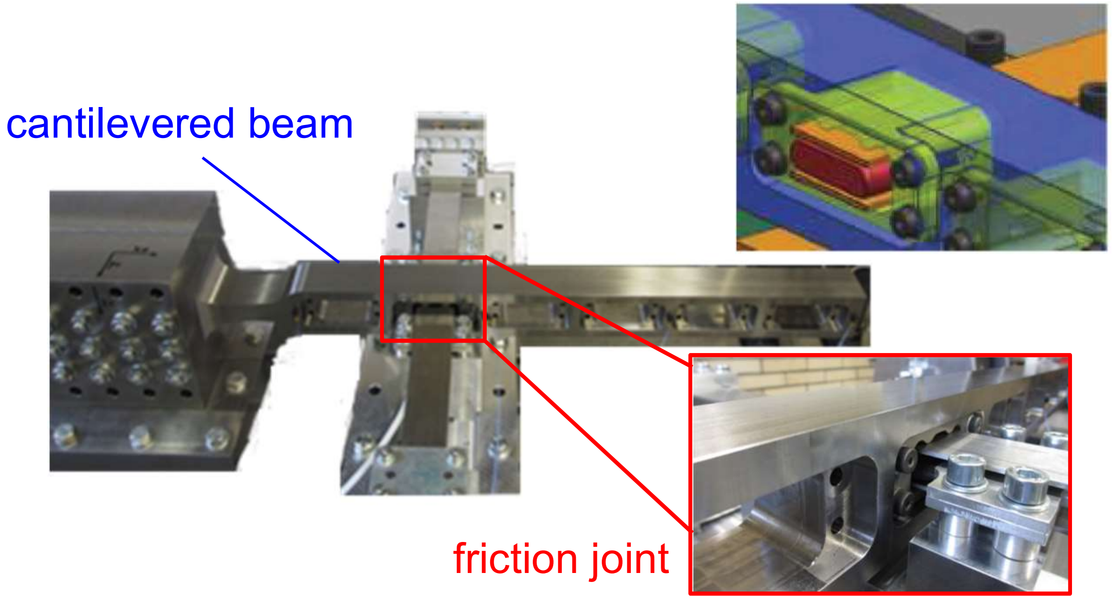

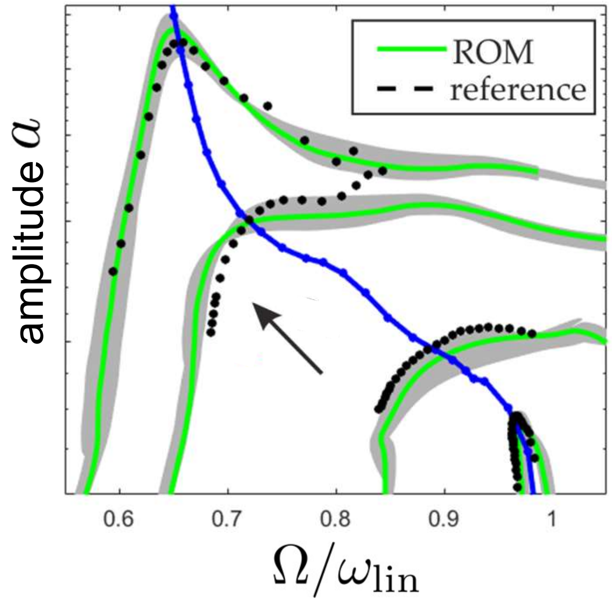

Benchmark specifically designed to challenge NSID methods

- excellent repeatability
- unprecedented frequency shift and variation of damping with amplitude
- significant change of deflection shape, including higher harmonics

[13] Scheel et al., JSV, 2020

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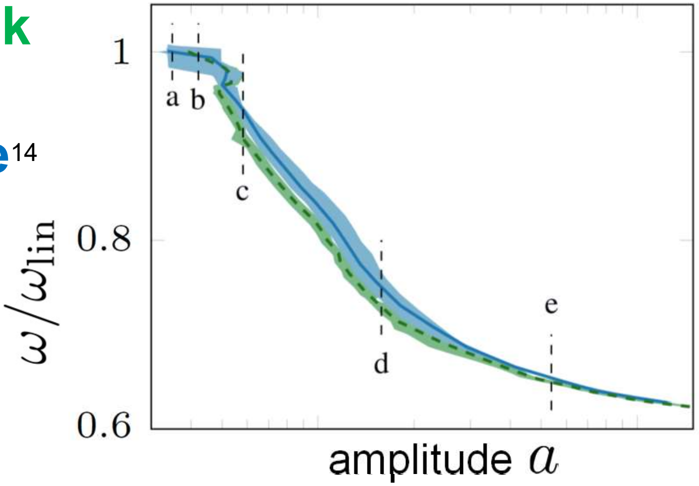
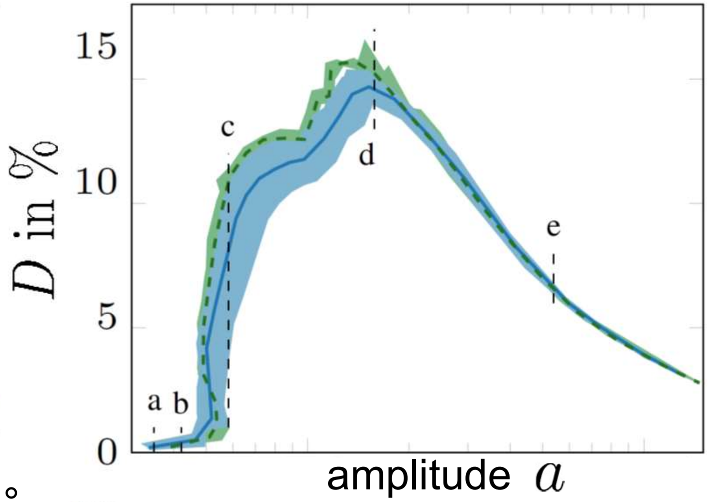
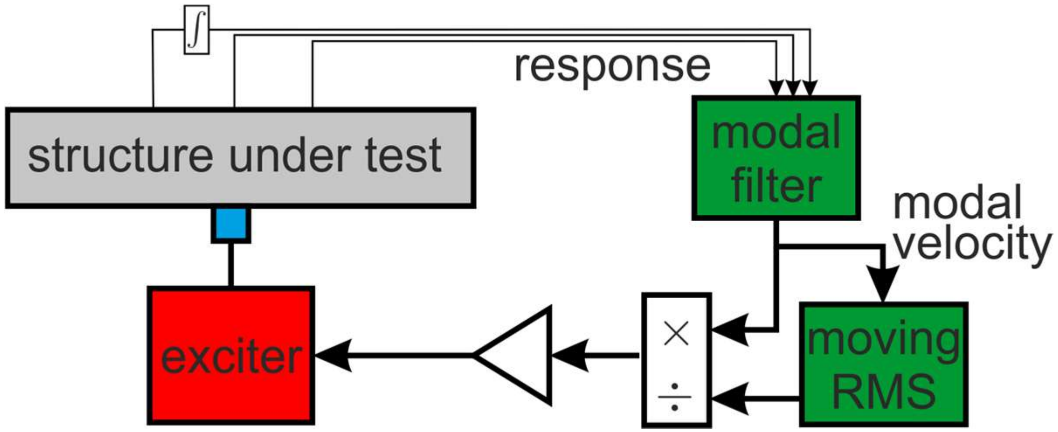
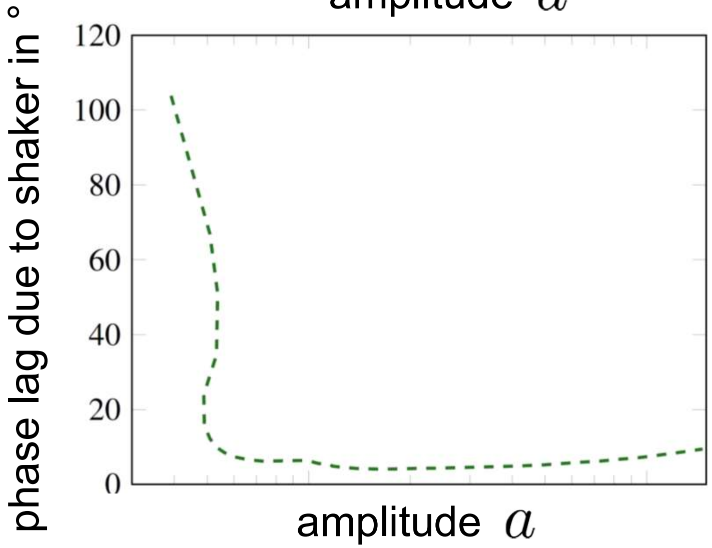
Velocity feedback
vs.
phase resonance[14]









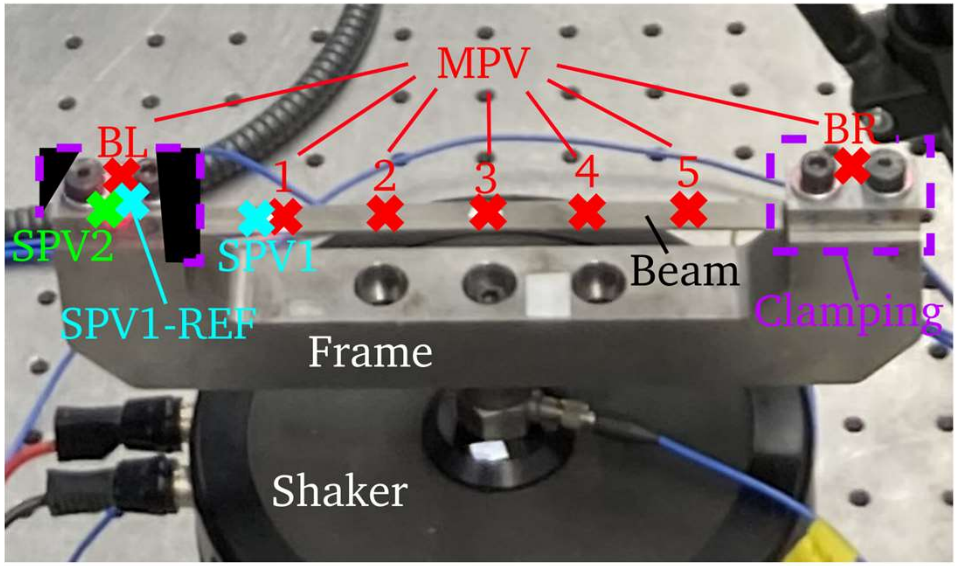
No feedback-control is needed, but new challenges arise!

- stability → normalize velocity (deliberate non-linearization)
- selection → apply modal filter; however, concentrated load yields gyroscopic forces
- phase lag → use more phase-preserving exciter?

[14] Scheel, MSSP, in review

Results: Clamped-clamped beam[15]

[15] Müller et al., MSSP, in preparation

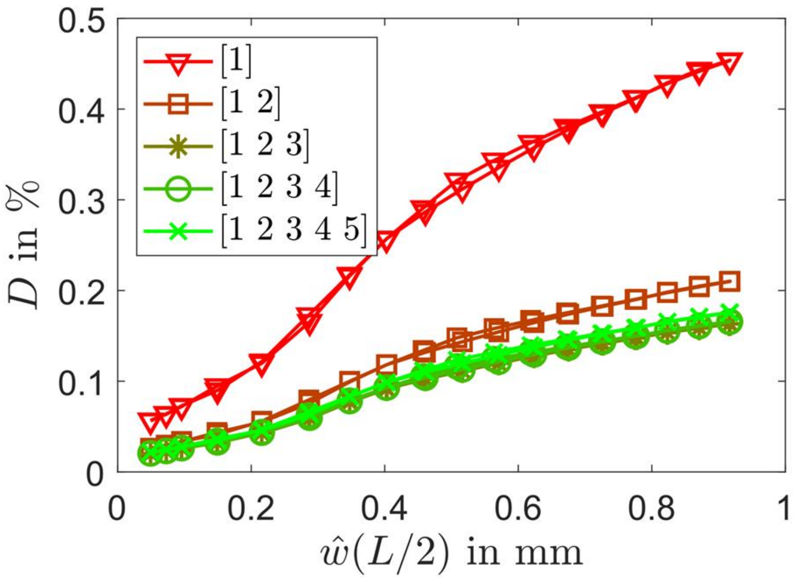


Base excitation:

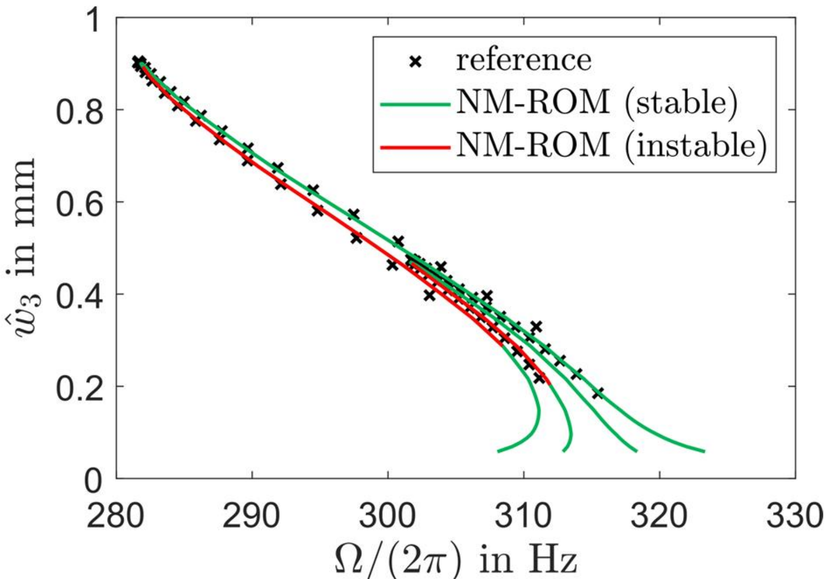
- one of the most popular variants of shaker testing (well-distributed load application; potential to reduce exciter-structure interaction)
- applied force impossible to measure
- to estimate power supplied by imposed inertia forces: measure response at multiple locations

$$\Rightarrow D = \frac{\hat{\boldsymbol{q}}_1^{\mathrm{H}} \boldsymbol{b} \hat{q}_{\mathrm{b}}}{2\|\hat{\boldsymbol{q}}_1\|^2}$$

splitting of absolute and relative displacement
$$\boldsymbol{q}_{\mathrm{abs}} = \boldsymbol{q} + \boldsymbol{b} q_{\mathrm{b}}$$



increasing damping typical for micro-slip



softening typical for initially arched beam

References

[1] Krack, M.: Nonlinear modal analysis of nonconservative systems: Extension of the periodic motion concept. Computers and Structures 154, 59-71, 2015. doi: 10.1016/j.compstruc.2015.03.008

[2] Jahn, M.; Tatzko, S.; Panning-von Scheidt, L.; Wallaschek, J.: Comparison of different harmonic balance based methodologies for computation of nonlinear modes of non-conservative mechanical systems. Mechanical Systems and Signal Processing 127, 159–171, 2019. DOI: 10.1016/j.ymssp.2019.03.005.

[3] Krack, M.: Extension of the single-nonlinear-mode theory by linear attachments and application to exciter-structure interaction. Journal of Sound and Vibration 505, 2021. doi: 10.1016/j.jsv.2021.116120

[4] Krack, M.; Panning-von Scheidt, L.; Wallaschek, J.: On the Computation of the Slow Dynamics of Nonlinear Modes of Mechanical Systems. Mechanical Systems and Signal Processing 42 (1-2), 71-87, 2013. doi: 10.1016/j.ymssp.2013.08.031

[5] B. Deaner, M. S. Allen, M. J. Starr, D. J. Segalman, and H. Sumali, "Application of Viscous and Iwan Modal Damping Models to Experimental Measurements From Bolted Structures," ASME Journal of Vibrations and Acoustics, vol. 137, p. 12, 2015.

[6] Link, M.; Boeswald, M.; Laborde, S.; Weiland, M.; Calvi, A.: An Approach to Non-linear Experimental Modal Analysis. Nonlinear Modeling and Applications, Volume 2: Proceedings of the 28th IMAC, A Conference on Structural Dynamics, 2010. New York, NY: Springer New York, S. 119–128. http://dx.doi.org/10.1007/978-1-4419-9719-7_11.

[7] Karaağaçlı, T.; Özgüven, H. N.: Experimental modal analysis of nonlinear systems by using response-controlled stepped-sine testing. Mechanical Systems and Signal Processing 146, S. 107023, 2021. DOI: 10.1016/j.ymssp.2020.107023.

[8] Renson, L.; Gonzalez-Buelga, A.; Barton D.; Neild, S.A.: Robust identification of backbone curves using control-based continuation. Journal of Sound and Vibration 367, 145–158, 2016.

[9] Kwarta, M.; Allen, M.S.: Nonlinear Normal Mode backbone estimation with near-resonant steady state inputs. Mechanical Systems and Signal Processing 162(1), 2022.

[10] Schwarz, S.; Kohlmann, L.; Hartung, A.; Gross, J.; Scheel, M.; Krack, M.: Validation of A Turbine Blade Component Test with Frictional Contacts by Phase-Locked-Loop and Force-Controlled Measurements. J. Eng. Gas Turbines Power, 2019. doi: 10.1115/1.4044772

[11] Scheel, M.; Peter, S.; Leine, R.I.; Krack, M.: A phase resonance approach for modal testing of structures with nonlinear dissipation. Journal of Sound and Vibration 435, 56-73, 2018. doi: 10.1016/j.jsv.2018.07.010

[12] Scheel, M.; Kleyman, G.; Tatar, A.; Brake, M.R.W.; Peter, S.; Noël, J.-P.; Allen, M.S.; Krack, M.: Experimental assessment of polynomial nonlinear state-space and nonlinear-mode models for near-resonant vibrations. Mechanical Systems and Signal Processing 143, 2020. doi: 10.1016/j.ymssp.2020.106796

[13] Scheel, M.; Weigele, T.; Krack, M.: Challenging an Experimental Nonlinear Modal Analysis Method with a New Strongly Friction-damped Structure. In review, Journal of Sound and Vibration.

[14] Scheel, M.: Nonlinear modal testing of damped structures: Velocity feedback vs. phase resonance. Mechanical Systems and Signal Processing, in review.

[15] Müller, F.; Woiwode, L.; Scheel, M.; Gross, J.; Krack, M.: Nonlinear Modal Testing using Base Excitation. In preparation.

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Conclusions

- The proposed methods are able to experimentally identify amplitude-dependent modal properties.
- The Single-Nonlinear-Mode Theory, along with recent extensions, permits powerful predictions in a large and technically relevant range of utility. It simplifies quantitative and qualitative analysis.
- The validation for a variety of benchmarks and nonlinearities confirms
  - the high accuracy of the identified modal properties (incl. damping!), and
  - the high robustness, simplicity and industrial maturity of the method.

Outlook

- Nonlinear Modal Testing
  – compensation of exciter-structure interaction via multi-harmonic control
  – systematic optimization of robustness and speed of control scheme
- Dynamic Substructuring based on Nonlinear Modes
- Application to Interdisciplinary Problems (FSI, VEH)