Reduced-order Modeling of the Loosening of Bolted Joints: Application to Axially Aligned Joints

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Motivation

- Loosening of bolts can be fatal and cause millions in damage.
  - 2011: loosening of single screw downed a $73 million UAV in service
  - 2017 survey: 29.1% of US playgrounds have loose bolts and are unsafe
  - 23% of all automobile service issues result from loose bolts
  - 20% of machine tool failures are caused by loose bolts.
  - Loose bolts rated top 10 health technology hazards

- While existing modeling approaches work for single bolts, they are simply intractable for multiple bolts.

Objectives:
1. Construct a reduced-order model (ROM) for bolted joint loosening that is accurate and computationally efficient.
2. Investigate and understand the interactions between multiple joints during loosening.

Paris train derailment in 2013 led to 6 deaths and dozens of injuries
Experimental Motivation

• Two hardened AISI 1566 steel rods connected with a threaded interface.

• Impacted by projectile fired by a gas-gun at 7, 10.8, and 13.5 m/s.

• Five initial torque preloads tested: 14, 30.5, 33.9, 41.5, and 54 Nm.

• Strains measured at four locations along each rod and interface filmed in some tests.

• Rotation measured using index after impact.

Reference
Experimental Results: Long-Time Results

- In long-time, response transitions from traveling waves to stationary waves (vibrations).
- Response strongly depends on the excitation amplitude:
  - For 54 Nm and 7 m/s, response is primarily linear.
  - For 54 Nm and 13.5 m/s, response is strongly nonlinear and joint loosens completely.
- WT reveal strong time (energy) dependence of participating harmonics.
Updated Linear Model Comparison

- Updated linear model reproduces measurement at 7 m/s.
- Updated linear model does not reproduce measurement at 13.5 m/s.
Proposed Reduced-order Model (ROM): Torque-Stiffness Relationship

- We define the stiffness of the joint to be a function of the torque: $k(T)$
- Joint stiffness has a physical maximum corresponding to stiffness of equivalent solid member:
  \[ k(T) = k_a \text{ for } T \geq T_m \]
  \[ k_a \equiv \text{maximum possible stiffness} \]
  \[ T_m \equiv \text{torque where max stiffness realized} \]
- In reality, maximum stiffness only realized if cold welding occurs, but the joint becomes inseparable.

- In a low-torque state, friction is weak and majority of work done increases stiffness of the joint. Mathematically,
  \[ \frac{dk}{dT} > 0 \text{ for } T = 0. \]
- In a high-torque state, friction is strong and minority of work done increases the stiffness of the joint. Mathematically,
  \[ \frac{dk}{dT} \to 0 \text{ as } T \to T_m. \]
Proposed Reduced-order Model (ROM): Torque-Stiffness Model

• One model that captures these observations is

\[ k(T) = k_a \left( 1 - \left( \frac{T}{T_m} \right)^\alpha \right)^\beta. \]

• Differentiating:

\[ \frac{dk}{dT} = \frac{\alpha \beta k_a}{T_m} \left( \frac{T}{T_m} \right)^{\alpha-1} \left( 1 - \left( \frac{T}{T_m} \right)^\alpha \right)^{\beta-1}. \]

• In a low-torque state, friction is weak and majority of work done increases stiffness of the joint. Thus, as \( T \to 0 \)

\[ \frac{dk}{dT} \to \alpha \beta k_a \left( \frac{T}{T_m} \right)^{\alpha-1} \text{ and if } \alpha > 1, \text{ then } \frac{dk}{dT} \to 0. \text{ Thus, we require that } [\alpha \leq 1] \]

• In a high-torque state, friction is strong and minority of work done increases the stiffness of the joint. Thus, as \( T \to T_m \)

\[ \frac{dk}{dT} \to \alpha \beta k_a \left( 1 - \left( \frac{T}{T_m} \right)^\alpha \right)^{\beta-1} \text{ and if } \beta < 1, \text{ then } \frac{dk}{dT} \to \infty \text{ and if } \beta = 1, \text{ then } \frac{dk}{dT} \to \frac{\alpha k_a}{T_m} \neq 0. \]

Thus, we require that \( \beta > 1 \).
Proposed Reduced-order Model (ROM): Torque Model

• Torque is dynamic and changes when the structure is excited.

• Experiments\(^1\) have achieved net loosening and net tightening.

• Experiments\(^2\) have shown that loosening occurs during loading and tightening during unloading.

• To capture the net loosening effect, we model the torque using a first-order ODE:

\[
\dot{T} + f(T) = G(t), T(0) = T_0.
\]

• In this work, we assume \(G(t) = 0\):

\[
\dot{T} + f(T) = 0, T(0) = T_0.
\]


Proposed Reduced-order Model (ROM): Torque Model

1. Torque only changes if there is relative motion across the joint: \( f(T) = f(T, z, \dot{z}), \quad z(t) = u_2(t) - u_1(t) \)

   Simple model: \( f(T, z, \dot{z}) = q(z, \dot{z})T \)

2. Bolt does not rotate when subjected to monotonic, quasi-static tensile loading.
   Thus, hypothesize that loss of torque is solely dependent on relative velocity:
   \[ q(z, \dot{z}) = q(\dot{z}). \]

3. To capture only loosening, rate of loss of torque independent of sign of relative velocity.

   Simple model: \( q(\dot{z}) = \gamma \dot{z}^2 \Rightarrow \dot{T} + \gamma \dot{z}^2T = 0, T(0) = T_0. \)

   - Governing equation for instantaneous torque:
     \[ \dot{T} + f(T) = 0, T(0) = T_0. \]
   - Functional form determined based on physical observations
• Torque-stiffness model:

\[ k(T) = k_a \left(1 - \left(\frac{T}{T_m}\right)^\alpha \right)^\beta. \]

• Parameters identified by curve fitting above equation to torque-stiffness pairs determined in previous study.

• Identified parameters:

\[
\begin{align*}
\alpha &= 0.7635, \\
\beta &= 1.9543, \\
T_m &= 77.48 \text{ Nm}.
\end{align*}
\]

• Torque ODE:

\[ \dot{T} + \gamma \dot{z}^2 T = 0, T(0) = T_0 \]

• Identify \( \gamma \) using instantaneous mean wavelet transform of experimental and predicted responses.

• Identified parameter:

\[ \gamma = 2690.6 \text{ s/m}^2 \]
Comparison of Proposed Model with Experiments

Linear Model: Strain Gage C, 54 Nm, 10.8 m/s

- Linear baseline model is unable to reproduce nonlinear response.
- Odd harmonics persist throughout response in model.

Updated Model: Strain Gage C, 54 Nm, 10.8 m/s

- Updated model with proposed ROM accurately reproduces dominant behavior.
- Odd harmonics exit response at similar times.
Validation of Proposed Model

- Updated model reproduces response when joint does not completely loosen.
- Measured torque loss: 34 Nm (63%)
  Model torque loss: 35.0 Nm (64%)

- Updated model with proposed ROM accurately reproduces dominant behavior.
- Odd harmonics exit response at earlier than in experiment.
**Objective:** Determine if and understand how multiple joints undergoing loosening can interact with each other.

**Approach:** Consider the behavior in 3-rod system with 2 axially aligned, threaded joints.

- All rods are uniform, homogeneous, and identical.
- Rods modeled using spectral element method.
- Square pulse applied to left boundary of leftmost rod:
  \[ F(t) = \begin{cases} 
  P & 0 < t \leq t_f \\
  0 & t > t_f 
  \end{cases}, \quad t_f = 73.2 \text{ ms} \]

- Threaded interface modeled using proposed approach:
  \[ K_i(T_i, \dot{z}_i) = k_a \left[ 1 - \left( 1 - \left( \frac{T_i}{T_m} \right)^\alpha \right)^\beta \right], i = 1, 2. \]

- Torques treated as new degrees of freedom:
  \[ \dot{T}_i + \gamma \dot{z}_i^2 T_2 = 0, \quad i = 1, 2, \]

- Joints may be torqued to different amounts.
• Natural frequencies vary based on the torque in each joint.

• Mode $3n + 1, n = 0,1,2 \ldots$ is unaffected by the joints.

• Transition in natural frequencies depends on the torque in each joint.
Simulation of Response: \( T_1(0) = T_2(0) = 54 \text{ Nm}, \ P = 160 \text{ kN} \)

- Strains start out the same, deviate after 0.08 seconds.
- Frequency transitions captured by WT.

- Rich frequency content transitions to motion in lowest flexible mode and rigid body mode.
Simulation of Response: $T_1(0) = T_2(0) = 54 \text{ Nm, } P = 160 \text{ kN}$

- Both joints appear to completely loosen.
- Comparable rates of loss of torque and stiffness.

$T_1(0) = 54 \text{ Nm}$
$T_2(0) = 54 \text{ Nm}$

$T_1(0.5) = 0 \text{ Nm}$
$T_2(0.5) = 2.9 \times 10^{-8} \text{ Nm}$

$K_1(0) = 8.53 \times 10^9 \text{ N/m}$
$K_2(0) = 8.53 \times 10^9 \text{ N/m}$

$K_1(0.5) = 0 \text{ N/m}$
$K_2(0.5) = 1131 \text{ N/m}$

- Stiffness of joint 2 effectively zero at end.
Simulation: $T_1(0) = 54$ Nm, $T_2(0) = 27$ Nm, $P = 160$ kN

- Strains start out similar but rapidly deviate.
- Rich frequency content observed before and after in Rods 1 & 2

- Rich frequency content transitions to motion in first two flexible modes in Rod 3.
Simulation: $T_1(0) = 54 \text{ Nm}, T_2(0) = 27 \text{ Nm}, P = 160 \text{ kN}$

- Unlike previous case, only joint 2 loosens completely.
- Comparable rates of loss initially.

- Rate of loss of torque in joint 1 changes abruptly after joint 2 loosens completely.
Simulation: $T_1(0) = 27$ Nm, $T_2(0) = 54$ Nm, $P = 160$ kN

- Like previous case, only joint 1 loosens completely.
- Joint 2 ends with higher torque than joint 1 in previous case.

- Rate of loss of torque in joint 2 changes abruptly after joint 1 loosens completely.
Simulation of Response with Different Initial Torques

- \( T_1(0) = T_2(0) = 54 \text{ Nm} \rightarrow \) Both loosen completely.

- \( T_1(0) = 54 \text{ Nm}, T_2(0) = 27 \text{ Nm} \rightarrow \) Only joint 2 loosens completely, different rates of loss.

- \( T_1(0) = 27 \text{ Nm}, T_2(0) = 54 \text{ Nm} \rightarrow \) Only joint 1 loosens completely, different rates of loss.

- In all cases, similar rate of loss until one joint loosens fully.

- Interested in understanding how the joints interact with each other.

- Simulate response for 10,000 combinations of torques (100 points for \( T_1(0), T_2(0) \in [14,54] \text{ Nm} \)).

- Measure remaining torque after 1 second of motion.

- Vary applied force to identify different regimes.
Regime I: Independence – $P = 50$ kN

- Torque remaining in each joint is independent of the initial torque of the other joint.
- Remaining torque in each joint is comparable.
Regime II: Weak Dependence - \( P = 80 \text{ kN} \)

- Torque remaining in each joint weakly depends on the initial torque of the other joint.
- Remaining torque in each joint is comparable.
Regime III: Strong Dependence - $P = 110$ kN

- Torque remaining in each joint is strongly depends on the initial torque of the other joint.
- Remaining torque in each joint is similar, but not the same.
- Starting to form bands of mitigation and band of loosening with same torque.
Regime IV: Bands of Mitigation – $P = 150 \text{ kN}$

- Torque remaining in each joint is strongly depends on the initial torque of the other joint.
- Bands of mitigation clearly visible for both joints, but at different initial torques.
Regime IV: Bands of Mitigation – \( P = 350 \text{ kN} \)

- Torque remaining in each joint is strongly depends on the initial torque of the other joint.
- Single band visible in each joint, but at different initial torques.
Regime V: Annihilation of Joint 1 \( - P = 500 \text{ kN} \)

- Torque remaining in each joint is strongly depends on the initial torque of the other joint.
- Joint 1 loosens completely regardless of initial torque. Joint 2 depends strongly on joint 1 initial torque.
Dynamics of split Hopkinson-pressure bar with threaded interface investigated subjected to axial shock excitation.

New ROM proposed for interface to capture loosening effects.

Resulting FE model reproduced the dominant effects of loosening.

Investigated behavior of three-rod, two-joint system.

Loosening of one joint can depend on the other joint.
  - Coupling through global dynamics

Bands of mitigation observed for certain loads.

Same initial tension often results in both joints loosening at relevant loads.

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